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An Estimated DSGE Model of Austria, the Euro Area and the U.S.: Some Welfare Implications of EMU*

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Abstract

We build a fully micro-founded dynamic general equilibrium (DSGE) model, which is estimated employing Bayesian methods. The model captures the most salient features of Austria as a small open economy, the Euro Area (EA) and the United States (U.S.). Further analysis is conducted through numerical simulations to examine how nominal and real shocks are propagated. Besides, welfare costs of nominal rigidities are calculated. We distinguish two sample periods, 'pre-EMU' and 'EMU'. In the former, we maintain the assumption of full commitment of respective (independent) Central Banks towards their monetary rules, whereas in the latter, the monetary policy of Austria is fully aligned with the European Central Bank.
Main results are derived from Bayesian estimation and simulation of the estimated model. Welfare calculations from the estimated model suggest that in the pre-EMU period, the EA and Austria present welfare costs close to one percent of steady-state consumption, whereas the U.S. welfare costs is slightly higher (-1.52 percent). As it would be expected, in the second subsample, welfare costs in the EA decrease, indicating an improvement in the allocation during the EMU regime (similarly in the U.S.), whereas in Austria welfare costs go up.

JEL classification: E42, F41, F42;
Keywords: Monetary policy, NOEM, DSGE models, Austria, Euro area, U.S.;

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An estimated DSGE model of Austria, the Euro Area and the U.S.: some welfare implications of EMU

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Abstract

We build a fully microfounded dynamic stochastic general equilibrium (DSGE) model, which is estimated employing Bayesian methods. The model captures the most salient features of Austria as a small open economy, the Euro Area (EA) and the United States (U.S.). Further analysis is conducted through numerical simulations to examine how nominal and real shocks are propagated. Besides, welfare costs of nominal rigidities are calculated. We distinguish two sample periods, 'pre-EMU' and 'EMU'. In the former, we maintain the assumption of full commitment of respective (independent) Central Banks towards their monetary rules, whereas in the latter, the monetary policy of Austria is fully aligned with the European Central Bank.

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Contents

1 Introduction

On 1 January 1999 the third and final stage of European Monetary Union (EMU) took place. That meant that 11 initial member states joined in a Monetary Union by delegating rights to manage their monetary policies on the European Central Bank (ECB). Greece became the 12th member of the EMU in 2001 and currently, the number of member states grew to 15. This had at least two results. First, nominal exchange rates of these member countries remained fixed since 1999. Second, though the Euro(€) started to circulate on 1 January 2002, since 1999 the ECB manages changes in the repo rate, which is the main monetary policy (MP) instrument.

The current research in macroeconomic has placed a considerable effort to extend closed economy macro-models to endogenize foreign economy fluctuations and at the same time to account for stylized features of the EMU as in Breuss & Rabitsch (2009), Pytlarczyk (2005), Benigno (2004), among others. This paper follows that direction by fully modeling Austria as an small open economy interacting with the EA and the U.S. as foreign economies, employing a consistent three-country dynamic stochastic general equilibrium (DSGE) model.[1]

The purpose of this paper is to estimate three countries' deep parameters, to seek whether there are substantial differences in 'pre-EMU' and 'EMU' regimes. In addition, we examine if those two regimes' estimates trigger meaningful welfare implications for Austria, Euro Area (EA) and the United States of America (U.S.).

Our model provides novel evidence on deep parameters for these countries using Bayesian estimation methods. These estimates are further utilized to parameterize a model that is useful to calculate endogenous variables' responses to various shocks and compare shocks' relative importance with Forecast Error Variance Decomposition (FEVD). Moreover, we seek evidence on welfare costs of different nominal rigidities in specific countries. In particular, given any shock, the idea is to check to which extent MP fulfils its stabilization role in Austria, in the EA and in the U.S.

The stabilization over the business cycle of output and consumption are crucial determinants of the welfare cost. Therefore, equipped with our model, this paper intends to respond to the following questions: What is the size of the welfare cost explained by nominal rigidities? And what is the welfare costs derived from firm's monopolistic power?

The distinction between 'pre-EMU' and 'EMU' regimes is very important. Breuss & Rabitsch (2009) reported different mode estimates for these periods, both for Austria and the EA.[2] For the U.S. case, there is ample evidence of a break in the MP management dated at the beginning of 1984. Smets & Wouters (2007) employ Bayesian methods to estimate a DSGE model and document two different modes that arise from two subsamples identified as 'Great Inflation' and 'Great Moderation'. These studies and stylized facts evidence from the data give us hint about subdividing the sample and to obtain more precise estimates.

In the literature, we find very few studies that build on three or more countries. For instance, Obstfeld & Rogoff (2005) have studied a model of three regions interpreting them as the U.S., Asia and Europe that disregards their production structures by assuming exogenously given endowments. Plasmans *et al.* (2006) propose a detailed modeling of a three country model, with a rich production side: final and intermediate goods (traded or nontraded). Our model draws on these studies and, especially, on the two-country model of Austria and the EA by Breuss & Rabitsch (2009).

We claim that three country models are more powerful and potentially more interesting than

[1] Although the present generation of DSGE models is not able to properly explain the outbreak of the present international financial crisis, it is a good instrument to study the implications of the political rescue measures (be it fiscal recovery packages or monetary easing) announced by many countries in 2008 and 2009. However, lack of data due to normal release delay prevents us from investigating crisis's effects.

[2] Mode estimates refer to the particular parameters' values that minimize the log-likelihood function, we expand on this later.

two-country models since we can study several transmission effects with more institutional realism.[3] The novelty of our three-country model lies in considering Austria is an open economy that interacts with the EA and the U.S.. Since in many aspects Austria is bounded by EA policy, it is potentially relevant for Austrian policymakers to understand the response of domestic variables to shocks with origin in the U.S. and assess if these responses would differ from those of the EA. To make more clear the point, note that though Austrian export shares to the U.S. and to the rest of the EA, on average represented 0.088 and 0.912 in the period 2000-2008, respectively, it is important to point out that shocks with origin in the U.S. would surely not be buffered (and so fully propagated into Austria) since the nominal exchange rate with respect to the rest of the EA members remains fixed.[4]

Our main findings come from analyzing the variance decomposition and welfare costs. First, the general conclusion is that domestic shocks matter a lot for output and consumption in Austria as well as in the foreign countries, in contrast foreign shocks are important for observed price inflation and observed nominal interest rates. Furthermore, wage inflation variability is also explained fundamentally by domestic shocks. Comparing consumption and investment variability, for Austria and the EA foreign shocks trigger more the former than the latter. Second, our estimates of welfare costs under the two regimes are similar to the obtained in the literature. For the pre-EMU period, EA and Austria present welfare costs close to one percent of steady-state consumption, whereas the U.S. welfare cost is above that number about -1.52 (we still report the negative sign since it suggests a cost). In the second subsample, the Austrian welfare cost is the only one that is magnified; in the case of the EA and the U.S. these drop to -0.83 and -0.98 percent, respectively.

The structure of this paper is as follow. In section 2, we present basic assumptions regarding the economic and institutional environment. Section 3 presents the DSGE model, covering consumer, government and firms problems, equilibrium conditions and the net foreign assets accumulation. Section 4 explains the welfare metric we apply. In Section 5 monetary and fiscal policy are specified, while Section 6 introduces driving exogenous shocks and the normalization of nominal endogenous variables. Section 7 deals with the data sources and with measurement assumptions. Section 8 discusses estimation results, while in Section 9 we concentrate in the salient features from the simulation exercise. Section 10 concludes and appendix A provides main derivations.

2 The economic framework

This paper assumes that the world economy comprises:

1. a small open economy like Austria,
2. a large country that represents the EA, the major trading partner for Austria,
3. the rest of the world, proxied (for simplicity) by the U.S.

These economies are populated with representative inhabitants that are small in comparison with the total population of the country. Our microfounded New Keynesian DSGE model provides us a suitable structure to analyze propagation mechanisms at work that may originate from worldwide (common) shocks or country-specific shocks. For instance, we may investigate how U.S.(EA)-specific shocks affect Austrian macroeconomic variables and how the responses develop

[3] For instance, we can closely follow relative prices and nominal exchange rates fluctuations when a monetary union is launched among two of these countries —e.g., large-large or large-small— or among the three.

[4] Note that 1 is assigned to the 'reduced' world comprising Austria, the rest of the EA and the U.S.. Data taken from United Nations COMTRADE database. For the sake of comparison, export shares of the rest of the EA to Austria and the U.S. are 0.252 and 0.748, respectively.

along time (which allow us to characterize propagation mechanisms at work). Further, we can provide insights on questions like: Which MP rule do minimize business cycle fluctuations generated from various shocks? How do asymmetric shocks influence aggregated prices and wages in Austria, the EA and the U.S.? According to latest figures from the International Monetary Fund, the U.S. GDP amounts to 23 percent of the worldwide GDP that would imply that whatever shock that hit the U.S. economy has tremendous impact the rest. Consequently, our answers to these questions are truly relevant for Austrian policymakers.

Before lying out the model structure, it is useful to comment on some institutional characteristics of the aforementioned economies. Beginning with Austria, we point out that the wage negotiation process is decentralized at the industry branch; however, there exists some sort of synchronization since the metal-workers' union is powerful enough to lead the annual negotiation, resulting in a new wage that is taken as a reference to emulate by other sectors. The Austrian National Bank (OeNB) manages independently the MP in the pre-EMU period.[5] Since 1999, the OeNB ceased to manage the MP which through its delegation to the ECB. Regarding fiscal policy, it remained more or less discretional, but as convergence to the EA deepened, its management approximated to an implicit zero-rule deficit. It became even more explicit the rule after the commitment to Stability and Growth Pact (SGP) which, in practice set upper bounds for the fiscal deficit and public debt.

The EA is treated as another country. Given the heterogeneity of the EA labor market, we assume that the aggregation effects reduce certain extreme wage setting cases such as, just to mention one, the wage indexation rule that operates in Belgium. Furthermore, in EA the fiscal and monetary policies are similarly managed as in Austria; however, the SGP can be thought of a rule that applies to both the EA as a country (national level) as well as its members (subnational levels). Finally, the nominal exchange rate w.r.t. the U.S. Dollar (USD) is fully flexible.

The U.S. proxies the rest of the world in our model. It is widely accepted that the U.S. labor market has a high degree of flexibility and absence of intervention from the government. Moreover, the literature in general states that the Central Bank of the U.S. (Fed) does not pursue an explicit inflation targeting rule; instead, it aims to reach a double-goal instead: price stability and full employment. Furthermore, it is hardly difficult to sustain that a fiscal rule is operative at the national level in the U.S.. However, balanced budget requirements are mandatory in many of the U.S. states.[6] For the U.S. case we just assume a zero deficit rule and we will argue why it is justified in Section 5.2.

To account for the wage stickiness consistent with institutional features mentioned above, it is assumed asynchronized negotiation of contracts which is rationalized as in Calvo (1983). Similarly, nominal prices are assumed to be sticky. The presence of rigidities is at the core of New Keynesian DSGE models.

We assume that the OeNB (at least in the Pre-EMU period), the ECB and the FED are autonomous in designing their monetary policies and they are fully committed to the announced rules. In the next section, we present a model that accounts for these characteristics.

3 The model

3.1 Consumer's problem

In this section we characterize and solve the consumer's intertemporal problem, where typically she is concerned with consumption-saving decisions that would have both present and future welfare

[5] Strictly, the MP launched a peg anchored to the Deutsche Mark since 1981.

[6] For instance, Canadian provinces are constrained by budgetary balance rules, but it is not the case at the national level.

implications. The second subsection examines the consumer's intratemporal problem, where she allocates consumption optimally so that the expenditure to buy varieties' bundles is minimized.

To begin with, we assume that the z-index identifies a particular country of the world economy. To make the presentation as general as possible, we employ set notation so that country z represents an element of the set $Z = \{AT, EA, US\}$, denoting Austria, the EA and the U.S., respectively.[7,8] In addition, Z^c denotes the complement set of set Z, where particular elements of Z^c are denoted with prime signs, i.e., z', z'', and so on. The complementary set is useful to simplify summations.

Regarding individuals that inhabit country z, we assume that there is a continuum of identical consumers denoted with the j^z-index, so that $j^z \in [0, n_z)$, where n_z stands for a share, $n_z \in (0,1)$, because the worldwide population is normalized to 1.[9] Notice that these shares are time invariant implying that no migration is assumed.

Throughout the presentation of the model, unless stated otherwise, the domestic agent j^z's optimization problem is presented, with the understanding that, unless stated otherwise, the foreign consumers make equivalent choices.

3.1.1 Intertemporal problem

Agent $j^z \in [0, n_z)$ has to choose how much he will spend today and how much tomorrow, given her willingness to wait or patience measured by $\beta \in (0,1)$. A key building bloc is the preference map conveyed in the period utility function, specified as:

$$U_t^{j^z} = \frac{\varepsilon_{U,t}^z}{1-\sigma_C^z}\left(C_t^{j^z} - h^z C_{t-1}^{j^z}\right)^{1-\sigma_C^z} + \varkappa_M^z \left(\frac{M_t^{j^z}}{P_{z,t}}\right)^{\sigma_M^z} - \frac{\varepsilon_{L,t}^z \varkappa_L^z}{1+\sigma_L^z}\left(L_t^{j^z}\right)^{1+\sigma_L^z}, \tag{1}$$

where σ_C^z is the coefficient of relative risk aversion, $\varkappa_M^z$ and $\varkappa_L^z$ are scale parameters, σ_M^z measures the elasticity of money demand and σ_L^z is the inverse of the Frisch elasticity, η^λ.[10] The following arguments enter in (1): consumption, real balances and labor effort. Consumption appears in its current as well as its lagged level altered by $0 < h^z < 1$ (*internal* habit formation).[11] In addition, the period utility depends positively on real balances weighted by $\varkappa_M^z$, while it depends negatively on the work effort exerted.[12] There are two shocks that distort Equation (1): $\varepsilon_{U,t}^z$ is a shock to consumption preferences centered on one with standard deviation equal to $\sigma_{\varepsilon_U^z}$ and $\varepsilon_{L,t}^z$ is a shock that distorts the equalization of the intratemporal rate of substitution between labor and consumption, specific to country z. A positive realization of $\varepsilon_{L,t}^z$ is interpreted as a pro-leisure shock, which is centered on one with constant standard deviation $\sigma_{\varepsilon_L^z}$.

We assume that a domestic labor bundler combines labor varieties efficiently —like a competitive intermediary— assuring labor market clearing. The bundler's demands of labor variety j^z, reads as:

$$L_t^{j^z} = \left(\frac{W_t^{j^z}}{W_{z,t}}\right)^{-\sigma_W^z} L_{z,t}, \tag{2}$$

[7] The EA is represented by 15 member states comprising Austria, Belgium, Cyprus, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Malta, Portugal, Slovenia, Spain and The Netherlands, whereas the U.S. is employed as a proxy for the world economy.

[8] The set Z is finite but it might have any dimension.

[9] The individual j^z is representative or average in the sense that she shares a common preferences map as well as a sequence of budget constraints. Hence, her choices (intertemporal and intratemporal) are similar as her neighbors' choices.

[10] The Frisch elasticity is defined as $\eta^\lambda \equiv \frac{U_L}{LU_{LL} - \frac{LU_{LC}^2}{U_{CC}}}$. Observe that for our separable CRRA utility function $\eta^{\lambda^z} = 1/\sigma_L^z$.

[11] The introduction of internal habit formation rather than external habit formation is motivated by the study of Grishchenko (2007). Using long-horizon aggregate stock market returns, she found that there is strong support for internal habit formation preferences, which decays slowly over time. In addition, this feature adds realism to the model's predictions (IRFs) and help explaining asset pricing puzzles. In particular, IRFs are much alike than those obtained with an unrestricted VAR.

[12] Strictly speaking, utility depends positively on leisure time; however, as it is common practice in the literature to avoid non-linearities, we specify it as a function that depends negatively on hours worked, where we normalize the full working day to one, i.e. 24 hours = 1.

where $W_t^{j^z}$ stands for the nominal wage asked by employee j^z, $W_{z,t}$ is the aggregate wage and $\sigma_W^z > 1$ is the constant elasticity of substitution (CES) between any pair of labor varieties. In Section 3.1.1 we make explicit how to obtain the numerator of the relative wage of Equation (2), whereas it can be shown that the denominator is:

$$W_{z,t} = \left[\left(\frac{1}{n_z}\right)\int_0^{n_z}\left(W_t^{j^z}\right)^{1-\sigma_W^z} dj^z\right]^{\frac{1}{1-\sigma_W^z}}. \tag{3}$$

As a result, substituting $L_t^{j^z}$ into Equation (1) yields:

$$U_t^{j^z} = \frac{\varepsilon_{U,t}^z}{1-\sigma_C^z}\left(C_t^{j^z} - h^z C_{t-1}^{j^z}\right)^{1-\sigma_C^z} + \varkappa_M^z\left(\frac{M_t^{j^z}}{P_{z,t}}\right)^{\sigma_M^z} - \frac{\varepsilon_{L,t}^z \varkappa_L^z}{1+\sigma_L^z}\left[\left(\frac{W_t^{j^z}}{W_{z,t}}\right)^{-\sigma_W^z} L_{z,t}\right]^{1+\sigma_L^z}, \tag{4}$$

where the wage $W_t^{j^z}$ is choice variable because it is assumed that the worker has monopolistic power over her particular abilities. Consumer j^z is confronted with two sequences of constraints (for $t, t+1, ...$). First, period budget constraints (CBC):

$$\begin{aligned} E_t\left[Q_z\left(s^{t+1},s^t\right)B_z^{j^z}\left(s^{t+1}\right)\right] + \sum_{z'\in Z^c} S_{z,z',t}E_t\left[Q_{z'}\left(s^{t+1},s^t\right)B_{z'}^{j^z}\left(s^{t+1}\right)\right] + M_t^{j^z} + P_{z,t}\left(C_t^{j^z} + I_t^{j^z}\right) \\ \leq B_z^{j^z}\left(s^t\right) + \sum_{z'\in Z^c} S_{z,z',t}B_{z'}^{j^z}\left(s^t\right) + M_{t-1}^{j^z} + \left(1-\tau_w^z\right)\mathcal{S}_W^z W_t^{j^z} L_t^{j^z} \\ +T_t^{j^z} - TX_t^{j^z} + \left(1-\tau_k^z\right)\left(\left(R_t^k\right)^{j^z} u_t^{j^z} K_{t-1}^{j^z} - P_{z,t}\Phi\left(u_t^{j^z}\right)K_{t-1}^{j^z}\right) + \mathsf{Pr}_t^{j^z}, \end{aligned} \tag{5}$$

where in the LHS we group consumer j^z's income allocations and in the RHS her sources. Beginning with the LHS, since assets markets are assumed to be complete, the outstanding domestic bonds holdings at the beginning of period $t+1$, $B_z^{j^z}\left(s^{t+1}\right)$, are valued at prices at the end of period t, $Q_z\left(s^{t+1},s^t\right)$, where s^t represents the state at end of period $t-1$. Likewise, foreign bonds are translated into domestic currency with the nominal exchange rates.[13] As usual, $E_t[\cdot]$ stands for the mathematical expectations operator, conditional on information set up to time t.

Complete assets markets have been studied quite a lot in the literature.[14] The basic result is that consumers will derive the same marginal utility of consumption and turning the discount factor $Q_z\left(s^{t+1},s^t\right)$ independent of j^z. This is valid under two conditions: (i) consumers have a 'common knowledge' of probabilities to get to s^{t+1}; and (ii) observed assets' prices are fair. Furthermore, among all assets issued in country z, there is an asset that is free of risk whose today's price is 1 and it will pay the return $r_{z,t}$ in the next period, regardless the state of nature. This leads to the following relationship:

$$R_{z,t}^{-1} = E_t\left[Q_z\left(s^{t+1},s^t\right)\right], \quad \forall z \in Z, \tag{6}$$

where $R_{z,t} \equiv 1 + r_{z,t}$, denotes country z's *gross* nominal interest rate.[15]

In the LHS of (5) we find nominal cash balances at the end of period t, $M_t^{j^z}$ and nominal private consumption plus investment expenditure. Moreover, there are beginning of period holdings of domestic, foreign bonds and money balances, the net of tax wage and net of tax capital income. Notice that $\mathcal{S}_W^z \geq 1$ is a wage subsidy if higher than one. The amount $\left(R_t^k\right)^{j^z} u_t^{j^z} K_{t-1}^{j^z} - P_{z,t}\Phi\left(u_t^{j^z}\right)K_{t-1}^{j^z}$

[13] From Austria's point of view, foreign bond holdings comprise: bonds issued in the EA, $B_{EA}^{j^{AT}}\left(s^{t+1}\right)$as well as bonds issued in the U.S., $B_{US}^{j^{AT}}\left(s^{t+1}\right)$. To convert into the domestic currency we employ the corresponding nominal exchange rates: $S_{AT,EA,t}$ and $S_{AT,US,t}$. Hence we get $\sum_{z'\in Z^c} S_{z,z',t}E_t\left[Q_{z'}\left(s^{t+1},s^t\right)B_{z'}^{j^z}\left(s^{t+1}\right)\right]$.

[14] For example, see Woodford (2003)'s model of the cashless economy.

[15] It is straightforward that (6) comes from $1 = E_t\left[Q_z\left(s^{t+1},s^t\right)\left(1+r_{z,t}\right)\right]$.

accounts for the nominal capital income and the function $\Phi(\cdot)$ penalizes over or under utilization of physical capital:

$$\Phi\left(u_t^{j^z}\right) = \frac{r_z^k}{\theta^z} \times \left\{\exp\left[\theta^z(u_t^{j^z} - 1)\right] - 1\right\}, \tag{7}$$

where θ^z is a shift parameter and r_z^k is the real steady state capital rental rate. Note that capital income simplifies to $\left(R_t^k\right)^{j^z} K_{t-1}^{j^z}$ when capital is fully utilized.[16] What is more, $\mathsf{Pr}_t^{j^z}$ stands for nominal profits that accrue to agent j^z, which are non-negative because prices exceed marginal costs, a common outcome from markets with monopolistic competition.[17] Second, the law of accumulation of physical capital reads as:

$$K_t^{j^z} = (1-\delta^z) K_{t-1}^{j^z} + \varepsilon_{I,t}^z I_t^{j^z} - \frac{\Psi^z}{2}\left(\frac{\varepsilon_{I,t}^z I_t^{j^z}}{K_{t-1}^{j^z}} - \delta^z\right)^2 K_{t-1}^{j^z}, \tag{8}$$

where $\varepsilon_{I,t}^z$ is an investment shock with mean one and (constant) standard deviation equal to $\sigma_{\varepsilon_I^z}$.

Consumer j^z's intertemporal problem is to maximize $\breve{U}_t^{j^z} \equiv E_0\left[\sum_{h=0}^{\infty}\beta^h U_{t+h}^{j^z}(.)\right]$, where $U_{t+h}^{j^z}$ is substituted by Equation (4), subject to a sequence of constraints (5) and (8).[18] The resulting first order conditions for an interior optimum, henceforth FOCs, w.r.t. consumption, money balances, domestic and foreign bonds, capital stock, investment and the rate of capital utilization (desired wages are presented in Section 3.1.1), are respectively as follows:

$$\Lambda_{z,t}^1 P_{z,t} = \varepsilon_{U,t}^z\left(C_t^{j^z} - h^z C_{t-1}^{j^z}\right)^{-\sigma_C^z} - \beta E_t\left[h^z \varepsilon_{U,t+1}^z\left(C_{t+1}^{j^z} - h^z C_t^{j^z}\right)^{-\sigma_C^z}\right], \tag{9}$$

where $\Lambda_{z,t}^1$ is the Lagrange multiplier associated to the nominal CBC, while the marginal utility w.r.t. consumption is $\tilde{\Lambda}_{z,t}^1 \equiv U_{C,t}^{j^z} = \Lambda_{z,t}^1 P_{z,t}$ (we omit the FOC w.r.t. $M_t^{j^z}$ since the CB manages the interest rate). The optimality condition w.r.t. domestic bonds is $\Lambda_{z,t}^1 E_t\left[Q_z\left(s^{t+1}, s^t\right)\right] = \beta E_t\left[\Lambda_{z,t+1}^1\right]$, which combined with Equation (9) yields the Euler condition:

$$\tilde{\Lambda}_{z,t}^1 = R_{z,t}\beta E_t\left[\frac{\tilde{\Lambda}_{z,t+1}^1}{\Pi_{z,t+1}}\right], \tag{10}$$

where gross price inflation definition, $\Pi_{z,t} \equiv \frac{P_{z,t}}{P_{z,t-1}}$, is employed. Similarly, the FOCs w.r.t. foreign bonds (for $\forall z' \in Z^c$) are:

$$\Lambda_{z,t}^1 S_{z,z',t} E_t\left[Q_{z'}\left(s^{t+1}, s^t\right)\right] = \beta E_t\left[\Lambda_{z,t+1}^1 S_{z,z',t+1}\right], \tag{11}$$

and replacing $\Lambda_{z,t}^1$ from Equation (9) and dividing by Equation (10) we get the following two UIP conditions (for $\forall z' \in Z^c$):

$$\frac{R_{z,t}}{R_{z',t}} = \frac{E_t\left[\tilde{\Lambda}_{z,t+1}^1 \frac{\Delta S_{z,z',t+1}}{\Pi_{z,t+1}}\right]}{E_t\left[\frac{\tilde{\Lambda}_{z,t+1}^1}{\Pi_{z,t+1}}\right]}, \tag{12}$$

where the gross depreciation rate is defined as $E_t\left[\Delta S_{z,z',t+1}\right] \equiv E_t\left[S_{z,z',t+1}/S_{z,z',t}\right]$.[19] Notice that the derivation of UIPs from the fundamental relationships of the model identifies the financial

[16] The FOC is $\Phi'(.) = r_z^k \exp[\theta^z(u_t^{j^z} - 1)]$. Because of symmetry of consumers, at the steady state $\Phi(u) = 0$ and $\Phi'(u) = r_z^k$.

[17] We define aggregate profits of firms located in country z as $\mathsf{Pr}_{z,t} \equiv \int_0^{n_z} \mathsf{Pr}_t^{i^z} di^z$ where the representative firm is indexed by i^z. Benefits that accrue in form of dividends to agent j^z are $\mathsf{Pr}_t^{j^z} \equiv \frac{\mathsf{Pr}_{z,t}}{n_z}$.

[18] Notice that at the equilibrium, Equation (5) is binding so that it is verified with equality.

[19] From the point of view of the foreign country, say country z', its counterpart UIP is: $\frac{R_{z',t}}{R_{z,t}} = \frac{E_t\left[\frac{\tilde{\Lambda}_{z,t+1}^1}{\Pi_{z',t+1}\Delta S_{z,z',t+1}}\right]}{E_t\left[\frac{\tilde{\Lambda}_{z,t+1}^1}{\Pi_{z',t+1}}\right]}$.

linkages that connect the three economies under study. If the parity of interest rates holds for any pair of countries, then there are no arbitrage opportunities arising from interest rates differentials (indeed under complete asset markets any interest rate differential is quickly offset by (an expected) currency depreciation).

The bilateral real exchange rate is defined as $RER_{z,z',t} \equiv \frac{S_{z,z',t}P_{z',t}}{P_{z,t}}$ ($\forall z' \in Z^c$). Following Chari *et al.* (2002), a formal expression for the real exchange rates result from solving an equality that combines the domestic (10) and foreign Euler conditions: $\frac{RER_{z,z',t+1}}{RER_{z,z',t}} = \frac{\tilde{\Lambda}^1_{z',t+1}/\tilde{\Lambda}^1_{z',t}}{\tilde{\Lambda}^1_{z,t+1}/\tilde{\Lambda}^1_{z,t}}$ and $\frac{RER_{z,z'',t+1}}{RER_{z,z'',t}} = \frac{\tilde{\Lambda}^1_{z'',t+1}/\tilde{\Lambda}^1_{z'',t}}{\tilde{\Lambda}^1_{z,t+1}/\tilde{\Lambda}^1_{z,t}}$. Iterating the latter backwards to period zero, yields:

$$RER_{z,z',t} = \kappa_1^z \frac{\tilde{\Lambda}^1_{z',t}}{\tilde{\Lambda}^1_{z,t}}, \quad \text{and} \quad RER_{z,z'',t} = \kappa_2^z \frac{\tilde{\Lambda}^1_{z'',t}}{\tilde{\Lambda}^1_{z,t}},$$

where κ_1^z and κ_2^z are constants that depend on initial conditions. Likewise, four additional bilateral real exchange rates formulae can be obtained; however, only three out of six bilateral real exchange rates are independent.

FOCs w.r.t. capital, investment and utilization rates can be written as:

$$\Lambda^2_{z,t} = \beta E_t \left\{ \begin{array}{c} \Lambda^1_{z,t+1}\left(1-\tau_k^z\right)\left(\left(R^k_{t+1}\right)^{j^z} u^{j^z}_{t+1} - P_{z,t+1}\Phi\left(u^{j^z}_{t+1}\right)\right) + \Lambda^2_{z,t+1} \\ \times \left[\left(1-\delta^z\right) - \frac{\Psi^z}{2}\left(\frac{\varepsilon^z_{I,t+1}I^{j^z}_{t+1}}{K^{j^z}_t} - \delta^z\right)^2 + \Psi^z\left(\frac{\varepsilon^z_{I,t+1}I^{j^z}_{t+1}}{K^{j^z}_t} - \delta^z\right)\frac{\varepsilon^z_{I,t+1}I^{j^z}_{t+1}}{K^{j^z}_t}\right] \end{array} \right\}, \tag{13}$$

$$\tilde{\Lambda}^1_{z,t} = \Lambda^2_{z,t} - \Lambda^2_{z,t}\Psi^z\left(\frac{\varepsilon^z_{I,t}I^{j^z}_t}{K^{j^z}_{t-1}} - \delta^z\right), \tag{14}$$

$$\left(R^k_t\right)^{j^z} = P_{z,t}\Phi'\left(u^{j^z}_t\right). \tag{15}$$

Finally, differentiation with respect to the Lagrange multiplier $\Lambda^2_{z,t}$, yields Equation (8).

Stickiness in nominal wages We assume that the consumer j^z supplies differentiated labor and enjoys monopolistic power in setting the nominal wage. Calvo (1983) assumes that once the wage is quoted, consumer j^z's supplied hours work meet firms' demand. In particular, Calvo wage setting assumes that a new wage is set by agent j^z once she is entitled to do so, and this allowance is materialized when she gets a random signal that follows an exogenous process. The occurrence of the signal is a probabilistic event with constant probability $\alpha^z_W \in (0,1)$. When the agent gets the signal, she chooses a wage, $\tilde{W}^{j^z}_t$, such utility (4) is optimized assuming that the nominal wage is maintained fixed until the signal is drawn again. Solving the problem's FOC w.r.t. $\tilde{W}^{j^z}_t$, it yields the following optimal nominal wage:

$$\left(\tilde{W}^{j^z}_t\right)^{1+\sigma^z_W\sigma^z_L} = \frac{\mu^z_{W,t}}{\left(1-\tau^z_w\right)\mathcal{S}^z_W} \frac{\mathrm{E}_t\left[\sum_{h=0}^{\infty}\left(\alpha^z_W\beta\right)^h \varepsilon^z_{L,t+h}\left(W_{z,t+h}\right)^{\sigma^z_W\left(1+\sigma^z_L\right)}\left(L_{z,t+h}\right)^{1+\sigma^z_L}\right]}{\mathrm{E}_t\left[\sum_{h=0}^{\infty}\left(\alpha^z_W\beta\right)^h \Lambda^1_{z,t+h}\left(W_{z,t+h}\right)^{\sigma^z_W} L_{z,t+h}\right]} \tag{16}$$

where $\mu^z_{W,t}$ can be interpreted as the desired time-varying markup and $\left(1-\tau^z_w\right)^{-1}$ is a constant that increases the wedge between $\tilde{W}^{j^z}_t$ and the marginal rate of substitution (MRS). Furthermore, $\mathcal{S}^z_W$

stands for a lump-sum subsidy —when greater than one— that (to some extent) may neutralize the markup and lead to a competitive equilibrium. Recently, Smets & Wouters (2007) have suggested that $\mu_{W,t}^{z}$ follows an exogenous law of motion, specified as ARMA(1,1) process which captures the very persistent wage markup's evolution. This modeling strategy has a clear advantage in the model's estimation stage: it is easier to identify μ_{W}^{z} than the elasticity σ_{W}^{z} because the *aggregate* data is not informative enough to capture the degree of substitution among labor varieties. Hence, a formal expression for the time-varying wage markup reads as:

$$\ln \mu_{W,t}^{z} = \left(1-\rho_{\mu_W}^{z}\right) \ln \mu_{W}^{z} + \rho_{\mu_W}^{z} \ln \mu_{W,t-1}^{z} + \nu_{W,t}^{z} - \vartheta_{W}^{z} \nu_{W,t-1}^{z},$$

where $\rho_{\mu_W}^{z}$ captures the persistence, whereas ϑ_{W}^{z} accounts for the memory of the wage markup shocks. Notice that μ_W is the (constant) long-run markup, $\sigma_{W}^{z}/(\sigma_{W}^{z}-1)$. Further, notice that the labor shock, $\varepsilon_{L,t}^{z}$, described above appears in the numerator. Expanding the summation in the numerator and denominator, introducing the expectations operator and assuming symmetry, it can be shown that (16) can be rewritten as:

$$\left(\tilde{W}_t\right)^{1+\sigma_W^z \sigma_L^z} = \frac{\mu_{W,t}^{z}}{\left(1-\tau_w^z\right) \mathcal{S}_W^z} \frac{\mathcal{W}_{1,z,t}}{\mathcal{W}_{2,z,t}}, \tag{17}$$

where the infinite summations are replaced by new variables:

$$\begin{aligned}
\mathcal{W}_{1,z,t} &\equiv \varepsilon_{L,t}^{z} W_{z,t}^{1+\sigma_W^z \sigma_L^z} L_{z,t}^{1+\sigma_L^z} + \alpha_W^z \beta \mathrm{E}_t \left[\mathcal{W}_{1,z,t+1}\right], \text{ and} \\
\mathcal{W}_{2,z,t} &\equiv \Lambda_{z,t}^{1} W_{z,t}^{\sigma_W^z} L_{z,t} + \alpha_W^z \beta \mathrm{E}_t \left[\mathcal{W}_{2,z,t+1}\right],
\end{aligned}$$

to exploit recursiveness.[20] Finally, notice that the observed domestic aggregate wage is a convex combination of the optimal wage determined by the Calvo rule and the wage quoted by those agents that were not able to reoptimize at time t (who keep their salaries fixed). Formally,

$$W_{z,t} = \left(1-\alpha_W^z\right) \tilde{W}_{z,t} + \alpha_W^z W_{z,t-1}. \tag{18}$$

3.1.2 Intratemporal optimization

To simplify notation, we drop the subindex t since it is implicit that the intratemporal optimization takes place at period t. Consumer j^z's nominal total private consumption expenditure equalizes the expenditure on home goods as well as imported goods: $P_z C^{j^z} = P_{z,z} C_z^{j^z} + \sum_{z' \in Z^c} P_{z,z'} C_{z'}^{j^z}$, an amount that the consumer seeks to minimize, given the following CES bundle definition:

$$C^{j^z} \equiv \left[\left(n_{z1}\right)^{\frac{1}{\eta^z}} \left(C_z^{j^z}\right)^{\frac{\eta^z-1}{\eta^z}} + \left(n_{z2}\right)^{\frac{1}{\eta^z}} \left(C_{z'}^{j^z}\right)^{\frac{\eta^z-1}{\eta^z}} + \left(1-n_{z1}-n_{z2}\right)^{\frac{1}{\eta^z}} \left(C_{z''}^{j^z}\right)^{\frac{\eta^z-1}{\eta^z}} \right]^{\frac{\eta^z}{\eta^z-1}}, \tag{19}$$

where $z \in Z = \{AT, EA, US\}$, while countries z' and z'' belong to Z^c. Notice that n_{z1} is the consumption share of home goods of country z, while n_{z2} and $(1-n_{z1}-n_{z2})$ are consumption shares of imported goods by country z, produced in countries z' and z'', respectively. We allow for different degrees home bias (or openness) in consumption for each country. In particular, home bias in consumption makes sense if $1 > n_{z2} > n_{z1} > \frac{1}{2}$ in Equation (19). Finally, the assumed elasticity of substitution between any pair of tradable goods is constant and equal to $\eta^z > 1$.[21]

In Appendix A.2.1 we work out the optimization problem. Because of the perfect competition assumption in intratemporal optimization problems, Lagrange multipliers associated with relevant

[20] Otherwise, infinite summations from Equation (16) cannot be handled by discrete algorithms.

[21] If $\eta^z \to 1$, then $C^{j^z} \equiv \frac{\left(C_z^{j^z}\right)^{n_{z1}} \left(C_{z'}^{j^z}\right)^{n_{z2}} \left(C_{z''}^{j^z}\right)^{1-n_{z1}-n_{z2}}}{n_{z1}^{n_{z1}} n_{z2}^{n_{z2}} \left(1-n_{z1}-n_{z2}\right)^{1-n_{z1}-n_{z2}}}$ and the true price index is $P_z = \left(P_{z,z}\right)^{n_{z1}} \left(P_{z,z'}\right)^{n_{z2}} \left(P_{z,z''}\right)^{1-n_{z1}-n_{z2}}$.

bundle definitions are identified with aggregate prices. In particular, country z's GDP deflator at time t is:

$$P_z^{1-\eta^z} \equiv n_{z1}\left(P_{z,z}\right)^{1-\eta^z} n_{z2}\left(P_{z,z'}\right)^{1-\eta^z}\left(1-n_{z1}-n_{z2}\right)\left(P_{z,z''}\right)^{1-\eta^z}.$$

The optimal intratemporal demands for domestic and foreign consumption aggregates are, respectively:

$$C_z^{j^z} = n_{z1}\left(\frac{P_{z,z}}{P_z}\right)^{-\eta^z} C^{j^z}, \tag{20}$$

$$C_{z'}^{j^z} = n_{z2}\left(\frac{P_{z,z'}}{P_z}\right)^{-\eta^z} C^{j^z}, \tag{21}$$

$$C_{z''}^{j^z} = \left(1-n_{z1}-n_{z2}\right)\left(\frac{P_{z,z''}}{P_z}\right)^{-\eta^z} C^{j^z}. \tag{22}$$

There will be nine demand functions. Moreover, in a three-country model, there are three bilateral *net* trade flows; however, two of them are independent because the world economy is closed.[22]

Next, given the consumption aggregates from Equations (20)-(22) agent j^z seeks to minimize expenditures $\int_0^{n_z} p(\zeta)C^{j^z}(\zeta)d\zeta$, by choosing $C^{j^z}(\zeta)$ (prices $p(\zeta)$ are parameters in the problem) subject to:

$$C_z^{j^z} \equiv \left[\left(\frac{1}{n_z}\right)^{\frac{1}{\varphi^z}} \int_0^{n_z} C^{j^z}(\zeta)^{\frac{\varphi^z-1}{\varphi^z}} d\zeta\right]^{\frac{\varphi^z}{\varphi^z-1}}, \tag{23}$$

where φ^z is the elasticity of substitution between varieties ζ produced in country $z \in Z$ and $\varphi^z > 1$.

Likewise, consumer j^z minimizes expenditures on imported goods.[23] Given the assumed structure of the world economy, there are two imports streams relevant for agent j^z to be minimized as well: $\int_0^{n_{z2}} p_z(\zeta')C^{j^z}(\zeta')d\zeta'$ and $\int_{n_{z1}+n_{z2}}^1 p_z(\zeta'')C^{j^z}(\zeta'')d\zeta''$, subject to import bundles similar to Equation (23). Solving the respective varieties' intratemporal problem, given the allocations $C_z^{j^z}$, $C_{z'}^{j^z}$ and $C_{z''}^{j^z}$, provides the following consumption demands of home and foreign varieties, respectively:

$$C^{j^z}(\zeta) = \frac{1}{n_z}\left(\frac{p(\zeta)}{P_{z,z}}\right)^{-\varphi^z} C_z^{j^z}, \tag{24}$$

$$C^{j^z}(\zeta') = \frac{1}{n_{z'}}\left(\frac{p_z(\zeta')}{P_{z,z'}}\right)^{-\varphi^z} C_{z'}^{j^z}, \tag{25}$$

$$C^{j^z}(\zeta'') = \frac{1}{1-n_z-n_{z'}}\left(\frac{p_z(\zeta'')}{P_{z,z''}}\right)^{-\varphi^z} C_{z''}^{j^z}. \tag{26}$$

where the typical variety is represented between brackets by an index (ζ, ζ' and ζ'') that accounts for the place where the manufacturing process takes place.[24] Finally, note that the implied Lagrange

[22] Likewise, there are three bilateral terms of trade and three bilateral real exchange rates; however, only two of them are independent. Formally, terms of trade are defined as: $T_{z,z'} \equiv \frac{P_{z,z'}}{P_{z,z}}$, $T_{z,z''} \equiv \frac{P_{z,z''}}{P_{z,z}}$ and $T_{z',z''} \equiv \frac{P_{z,z''}}{P_{z,z'}} = \frac{T_{z,z''}}{T_{z,z'}}$.

[23] Note that we follow the same logic to identify typical varieties, i.e., ζ' and ζ'' are typical varieties produced in countries z' and z'', both of them belong to Z^c.

[24] In the absence of transaction costs, then the law of one price (LOOP) would apply. Thus, $p_z(\zeta') = \frac{p(\zeta')}{S_{z',z}}$ as well as $p_z(\zeta'') = \frac{p(\zeta'')}{S_{z'',z}}$.

multipliers are the prevailing prices for the corresponding aggregates, which are defined as follow:

$$P_{z,z}^{1-\varphi^z} \equiv \left(\frac{1}{n_z}\right)^{\frac{1}{\varphi^z}} \int_0^{n_z} p(\zeta)^{1-\varphi^z} d\zeta, \tag{27}$$

$$P_{z,z'}^{1-\varphi^z} \equiv \left(\frac{1}{n_{z'}}\right)^{\frac{1}{\varphi^z}} \int_0^{n_{z'}} p_z(\zeta')^{1-\varphi^z} d\zeta',$$

$$P_{z,z''}^{1-\varphi^z} \equiv \left(\frac{1}{1-n_z-n_{z'}}\right)^{\frac{1}{\varphi^z}} \int_{n_z+n_{z'}}^1 p_z(\zeta'')^{1-\varphi^z} d\zeta''.$$

Demands for investment goods are similarly derived as consumption demands. The nominal total private investment expenditure is: $P_z I^{j^z} = P_z I_z^{j^z} + \sum_{z' \in Z^c} P_{z,z'} I_{z'}^{j^z}$, which agent j^z seeks to minimize given the following Armington investment bundle (cfr. Equation (19)):

$$I^{j^z} \equiv \left[(n_{z1})^{\frac{1}{\eta^z}} \left(I_z^{j^z}\right)^{\frac{\eta^z-1}{\eta^z}} + (n_{z2})^{\frac{1}{\eta^z}} \left(I_{z'}^{j^z}\right)^{\frac{\eta^z-1}{\eta^z}} + (1-n_{z1}-n_{z2})^{\frac{1}{\eta^z}} \left(I_{z''}^{j^z}\right)^{\frac{\eta^z-1}{\eta^z}} \right]^{\frac{\eta^z}{\eta^z-1}}. \tag{28}$$

Proceeding in the same manner as with consumption demands, home and imported investment demands (crf. Equations (20)-(22)) are:

$$I_z^{j^z} = n_{z1} \left(\frac{P_{z,z}}{P_z}\right)^{-\eta^z} I^{j^z}, \tag{29}$$

$$I_{z'}^{j^z} = n_{z2} \left(\frac{P_{z,z'}}{P_z}\right)^{-\eta^z} I^{j^z}, \tag{30}$$

$$I_{z''}^{j^z} = (1-n_{z1}-n_{z2}) \left(\frac{P_{z,z''}}{P_z}\right)^{-\eta^z} I^{j^z}. \tag{31}$$

Given investment aggregate indices $I_z^{j^z}$, $I_{z'}^{j^z}$ and $I_{z''}^{j^z}$, corresponding home and imported investment goods varieties are, respectively:

$$I^{j^z}(\zeta) = \frac{1}{n_z} \left(\frac{p(\zeta)}{P_{z,z}}\right)^{-\varphi^z} I_z^{j^z}, \tag{32}$$

$$I^{j^z}(\zeta') = \frac{1}{n_{z'}} \left(\frac{p_z(\zeta')}{P_{z,z'}}\right)^{-\varphi^z} I_{z'}^{j^z}, \tag{33}$$

$$I^{j^z}(\zeta'') = \frac{1}{1-n_z-n_{z'}} \left(\frac{p_z(\zeta'')}{P_{z,z''}}\right)^{-\varphi^z} I_{z''}^{j^z}. \tag{34}$$

3.2 Government

The government solves a simple intratemporal problem to allocate its resources. The issue is how to allocate the total nominal public expenditure $P_z G_z = \int_0^{n_{z1}} p(\zeta) g(\zeta) d\zeta$ by choosing optimally $g(\zeta)$. Solving, we obtain the domestic public demand of variety ζ:

$$g(\zeta) = \left(\frac{p(\zeta)}{P_z}\right)^{-\varphi^z} G_z, \tag{35}$$

where G_z is scaled to the GDP following the rule:

$$G_{z,t} = \left(1 - \frac{1}{g_{z,t}}\right) Y_{z,t}, \tag{36}$$

where $\left(1-\frac{1}{g_{z,t}}\right)$ is a stochastic factor driven by $g_{z,t}$, which has a data generating process that reads as:

$$\ln g_{z,t} = (1-\rho_G^z)\ln g_z + \rho_G^z \ln g_{z,t-1} + \varepsilon_{G,t}^z, \tag{37}$$

where g_z is calibrated so that the ratio G_z/Y_z matches the data. Further, $\varepsilon_{G,t}^z$ is a shock with zero mean and constant standard deviation $\sigma_{\varepsilon_G^z}$.

The government is bound by the so-called government budget constraint (GBC), which states that the genuine resources from the government must be enough to finance its expenses at any period t. Formally,

$$\begin{aligned} TX_{z,t} + M_{z,t} - M_{z,t-1} + E_t\left[Q_z\left(s^{t+1},s^t\right)B_z^{j^z}\left(s^{t+1}\right)\right] - B_z^{j^z}\left(s^t\right) \\ +\sum_{z'\in Z^c}\left\{E_t\left[Q_z\left(s^{t+1},s^t\right)B_z^{j^{z'}}\left(s^{t+1}\right)\right] - B_z^{j^{z'}}\left(s^t\right)\right\} \geqslant T_{z,t} \\ +P_{z,t}G_{z,t} + \left(\mathcal{S}_P^z - 1\right)\int_0^{n_z}\tilde{p}_t^{i^z}(\zeta)Y_{z,t+h}^{i^z}di^z + \left(\mathcal{S}_W^z-1\right)\int_0^{n_z}W_t^{j^z}L_t^{j^z}dj^z, \end{aligned} \tag{38}$$

where $TX_{z,t}$ stands for the genuine taxes levied from various sources which vary over the business cycle. Formally,

$$TX_{z,t} \equiv \tau_w^z W_{z,t}L_{z,t} + \tau_k^z\left[R_{z,t}u_{z,t}K_{z,t-1} - P_{z,t}\Phi\left(u_{z,t}\right)K_{z,t-1}\right] + IT_{z,t}, \tag{39}$$

where the income tax levied on firms is defined as $IT_{z,t} \equiv \tau_I^z[\ n_z P_{z,z,t}\left(C_{z,z,t}+I_{z,z,t}+G_{z,z,t}\right) + n_{z'}\frac{P_{z',z,t}}{S_{z,z',t}}\left(C_{z',z,t}+I_{z',z,t}\right)\ +\ (1-n_z-n_{z'})\ \frac{P_{z'',z,t}}{S_{z,z'',t}}\ \left(C_{z'',z,t}+I_{z'',z,t}\right)]$ and τ_w^z, τ_k^z and τ_I^z are (marginal and average) tax rates applied on salary income, capital income and firm revenue, respectively. Additional sources of government income is the net issuance of money and bonds denominated in domestic currency, also sold to foreign agents $j^{z'}(z'\in Z^c)$. Integrating, the gross external debt equalizes $\int_0^{n_z}B_z^{j^z}\left(s^{t+1}\right)dj^z + \int_0^{n_{z'}}B_z^{j^{z'}}\left(s^{t+1}\right)dj^{z'} + \int_{n_z+n_{z'}}^1 B_z^{j^{z''}}\left(s^{t+1}\right)dj^{z''}$. In addition, $(\mathcal{S}_P^z-1)$ and $(\mathcal{S}_W^z-1)$ are net subsidies to the private sector to neutralize the price and wage markups, respectively. Finally, $T_{z,t}$ stands for transfers (or lump-sum taxes if negative) to the public.

3.3 Firm's problem

We solve the firm i^z's problem in two stages. First, we solve the intratemporal problem that assures allocative efficiency of resources (inputs). Second, we introduce the institutional framework that conditions the pricing policy of the company, such that it maximizes the present value of the future stream of profits. Foreign firms solve equivalent economic problems.

3.3.1 Intratemporal problem: efficiency

Consider a representative firm i^z established in country z, which is fully specialized in the production of variety ζ. To carry out the production process, it hires capital services $(u_{z,t}K_{t-1}^{i^z})$ and labor to efficiently produce $Y_t^{i^z}(\zeta)$ according to the following Cobb-Douglas production function that includes a fixed cost of running the firm:

$$Y_t^{i^z}(\zeta) \equiv A_{z,t}\left(u_{z,t}K_{t-1}^{i^z}\right)^{\alpha^z}\left(L_t^{i^z}\right)^{1-\alpha^z} - FC_z, \tag{40}$$

where $0<\alpha^z<1$.[25] A_t denotes total factor productivity, while $K_{t-1}^{i^z}$ and $L_t^{i^z}$ stand for the physical capital stock at the beginning of the period t and aggregate labor effectively demanded by firm

[25] As $\alpha^z$is constant this implies that the income distribution is kept fixed throughout the analysis.

i^z, respectively.[26] Technology displays decreasing marginal returns on both labor and capital and positive cross-marginal productivity. Although the multiplicative part of the production function is homogeneous of degree one, the addition of the fixed cost turns returns to scale in technology increasing, see Schmitt-Grohe & Uribe (2005).

The technological shock follows an exogenous process defined as:

$$\ln(A_{z,t}/A_z) \equiv \rho_A^z \ln(A_{z,t-1}/A_z) + \varepsilon_{Y,t}^z, \tag{41}$$

where $\varepsilon_{Y,t}^z$ is a shock with mean zero and standard deviation $\sigma_{\varepsilon_Y^z}$, whereas A_z measures the long-run (constant) technology level.

Firm i^z is pricetaker in the inputs market, taking as given $R_{z,t}^k$ and $W_{z,t}$ per unit of time. Moreover, it is assumed that all workers provide labor to each firm, being this information gathered in the following aggregate:

$$L_t^{i^z} \equiv \left[\left(\frac{1}{n_z} \right)^{\frac{1}{\sigma_W^z}} \int_0^{n_z} \left(L_t^{i^z, j^z} \right)^{\frac{\sigma_W^z - 1}{\sigma_W^z}} dj^z \right]^{\frac{\sigma_W^z}{\sigma_W^z - 1}}, \tag{42}$$

where $L_t^{i^z, j^z}$ is the number of hours worked by agent j^z at firm i^z. Given the technology, firm i^z's problem can be stated as:

$$\min_{\left\{L_t^{i^z}, K_{t-1}^{i^z}\right\}} W_{z,t} L_t^{i^z} + u_{z,t} R_{z,t}^k K_{t-1}^{i^z}, \quad \text{s.t. Equation (40).}$$

Optimality conditions lead to the equalization of the technical rate of substitution to the relative inputs price (or economic rate of substitution):

$$\frac{1 - \alpha^z}{\alpha^z} = \frac{W_{z,t} L_t^{i^z}}{R_{z,t}^k u_{z,t} K_{t-1}^{i^z}}, \tag{43}$$

where since the LHS ratio is a constant, it follows that all firms established in country z —given the common knowledge of the technology— have the same technical rate of substitution. Resulting optimal inputs' demands are substituted back into the total cost function, from where we derive the following (nominal) marginal cost function:[27]

$$MC_{z,t} = \frac{1}{A_z} \frac{\left(R_{z,t}^k\right)^{\alpha^z} \left(W_{z,t}\right)^{1-\alpha^z}}{\left(\alpha^z\right)^{\alpha^z} \left(1 - \alpha^z\right)^{1-\alpha^z}}, \tag{44}$$

which depends positively on nominal wages and capital rental rate and negatively on technology. Further, notice that $MC_{z,t}$ is independent of the production level due to two assumptions: (i) Cobb-Douglas technology coupled with constant returns to scale; and (ii) the augmenting productivity process defined in (41) is assumed to be 'common' knowledge.

3.3.2 Firm's pricing

This section reviews the pricing strategy followed by firms that serve the domestic and foreign markets. To simplify matters, we assume that either the firm targets the home market or exports its production, and in any case, it specializes in the production of one variety.

[26] Notice that the technology process is a public good in country z and fixed costs are similar for all firms, therefore we drop the upper-index i^z.

[27] The marginal cost is the partial derivative of the total cost function w.r.t. $Y_t^{i^z}$.

Beginning with those firms that sell their production at home, we obtain the demand by combining domestic demands of variety ζ from Equations (24), (32) and (35) yield an aggregated demand of variety ζ, $Y_d^{j^z}(\zeta)$, as follows:

$$Y_d^{j^z}(\zeta) = \left(\frac{p(\zeta)}{P_{z,z}}\right)^{-\varphi^z} \left[\frac{n_{z1}}{n_z}\left(\frac{P_{z,z}}{P_z}\right)^{-\eta^z}\left(C^{j^z} + I^{j^z}\right) + \left(\frac{P_{z,z}}{P_z}\right)^{-\varphi^z} G_z\right], \tag{45}$$

which ignores exports to countries that belong to Z^c.[28]

Foreign demands, $Y_d^{j^{z'}}(\zeta)$ and $Y_d^{j^{z''}}(\zeta)$, can be easily formulated given Equations (20) and (29). We assume that the firm has no power to price differently if they serve other markets than the domestic; hence, it follows that adding up $Y_d^{j^z}(\zeta)$, $Y_d^{j^{z'}}(\zeta)$ and $Y_d^{j^{z''}}(\zeta)$ yield the global domestic variety ζ's demand which is equal to the firm i^z's output, $Y_t^{i^z}(\zeta) + FC_z$, according to (40).

Following Calvo (1983), firms are allowed to change prices when receiving a random signal. Thus, changing the price is an event with constant probability of occurrence, $(1-\alpha_P^z)$, which is known and independent of previous (historic) draws. Under the hypothesis that firm i^z is not allowed to reset prices in future periods, it seeks to maximize the present value of the stream of future profits, $\mathsf{Pr}_t^{i^z}$, subject to the fact that it serves variety ζ's aggregated demand (or, as we stated above $Y_t^{i^z}(\zeta) + FC_z$). Formally, firm i^z's present value benefits results in the following unconstrained problem:

$$\mathsf{Pr}_t^{i^z} = E_t\left[\sum_{h=0}^{\infty}(\alpha_P^z\beta)^h \frac{\Lambda_{z,t+h}^1}{\Lambda_{z,t}^1}\left[(1-\tau_I^z)\,\mathcal{S}_P^z\tilde{p}_t^{i^z}(\zeta)Y_{z,t+h}^{i^z}(\zeta) - TC_{z,t+h}(Y_{z,t+h}^{i^z}(\zeta) + FC_z)\right]\right], \tag{46}$$

where τ_I^z is the tax rate on firm i^z's income.

Since all profits are distributed through dividends, the consumer's relevant discount factor equals $\beta^h E_t\left[\Lambda_{z,t+h}^1\right]/\Lambda_{z,t}^1$, which is j^z-independent because assets markets are complete. Solving, the optimal price for the consumption variety that results is:

$$\tilde{p}_t^{i^z}(\zeta) = \frac{\mu_{P,t}^z}{(1-\tau_I^z)\,\mathcal{S}_P^z}\frac{E_t\left[\sum_{h=0}^{\infty}(\alpha_P^z\beta)^h\,\Lambda_{z,t+h}^1 MC_{z,t+h}\left(Y_{z,t+h}^{i^z}(\zeta) + FC_z\right)\right]}{E_t\left[\sum_{h=0}^{\infty}(\alpha_P^z\beta)^h\,\Lambda_{z,t+h}^1\left(Y_{z,t+h}^{i^z}(\zeta) + FC_z\right)\right]}, \tag{47}$$

where we define the long run markup as $\mu_P^z \equiv \frac{\varphi^z}{(\varphi^z-1)}$ and $\mathcal{S}_P^z$ stands for a lump-sum subsidy —when greater than one— that (to some extent) neutralizes the markup. Similarly as with wages, Smets & Wouters (2007) have suggested that $\mu_{P,t}$ follows an ARMA(1,1) process that reads as:

$$\ln\mu_{P,t}^z = \left(1-\rho_{\mu_W}^z\right)\ln\mu_P^z + \rho_{\mu_P}^z\ln\mu_{P,t-1}^z + \nu_{P,t}^z - \vartheta_P^z\nu_{P,t-1}^z,$$

where $\rho_{\mu_P}^z$ capture the persistence while ϑ_P^z captures the memory of price shocks processes. Recall that $MC_{z,t+h}$ are not firm specific.

As Equation (47) contains infinite summations, we seek a recursive form. Again, expanding the summation in the numerator and in the denominator, introducing the expectations operator, it is possible to show that Equation (47) can be equally written as:[29]

$$\tilde{p}_t^{i^z}(\zeta) = \frac{\mu_{P,t}^z}{(1-\tau_I^z)\,\mathcal{S}^z}\frac{\mathcal{P}_{1,z,t}}{\mathcal{P}_{2,z,t}}, \tag{48}$$

[28] Notice that $\left(\frac{p(\zeta)}{P_{z,z}}\right)^{-\varphi^z}$ is factored out of the brackets since the private sector as well as the government varieties' demands have elasticities of substitution equal to φ^z.

[29] Note the parallelism in the analysis that leaded us to redefine variables in Equation (17). See footnote 20.

where the numerator and the denominator of (47) are replaced by the following variables to exploit formulae recursiveness:

$$\mathcal{P}_{1,z,t} \equiv \Lambda^1_{z,t} MC_{z,t} \left(Y^{i^z}_{z,t}(\zeta) + FC_z \right) + \alpha^z_P \beta \mathrm{E}_t \left[\mathcal{P}_{1,z,t+1} \right] \tag{49}$$

$$\mathcal{P}_{2,z,t} \equiv \Lambda^1_{z,t} \left(Y^{i^z}_{z,t}(\zeta) + FC_z \right) + \alpha^z_P \beta \mathrm{E}_t \left[\mathcal{P}_{2,z,t+1} \right] \tag{50}$$

The observed domestic price for home consumption goods is a convex combination of the optimal price determined by the Calvo price rule, Equation (47), and the price quoted by those agents that were not able to reoptimize at time t. We drop i^z in (48) due to symmetry, and taking into account (27) it follows that the aggregate price reads as:

$$P^{1-\varphi^z}_{z,z,t} = (1 - \alpha^z_P)\, \tilde{p}_{z,t}(\zeta)^{1-\varphi^z} + \alpha^z_P P^{1-\varphi^z}_{z,z,t-1}. \tag{51}$$

We assume that exporters quote prices in exporters' currency and that the law of one price (LOOP) holds.[30]

3.4 Equilibrium conditions

In this section we elaborate on the aggregation of the macro-variables. First, we derive the aggregate demand in the goods markets. Second, we impose equilibrium in the inputs market.

3.4.1 Aggregate demand

Consider the nominal gross domestic product (GDP) of the home economy, defined as the product of the real GDP and the GDP deflator. Formally,

$$GDP_{z,t} \equiv P_{z,t} gdp_{z,t} = P_{z,t} Y^d_{z,t}, \tag{52}$$

where $Y^d_{z,t} \equiv C_{z,t} + G_{z,t} + I_{z,t} + \Phi\left(u_{z,t}\right) K_{z,t-1}$. The latter represents the country z's aggregate demand which comprises the sum of (deflated) domestic demands for private and government consumption, investment, foreign demand of home goods (for consumption and investment motives) minus what is lost because of the adjustment in capital utilization, $\Phi\left(u_{z,t}\right) K_{z,t-1}$.

A more detailed examination at $Y^d_{z,t}$ results from disaggregating it along the following two aggregation dimensions. This is done in the following subsections.

Integration among agents Adding up agent j's demands for any generic variety ζ —agent j stands for any agent of the world economy— yields $Y^d(\zeta)$, which depends on contemporary flow variables. Next, we build an expression for $Y^d(\zeta)$ integrating over agents.

Beginning with private demand, it follows that worldwide consumption and investment of the home variety ζ can be written as:

$$\begin{aligned} \int_0^1 \left[C^j(\zeta) + I^j(\zeta) \right] dj = {} & n_{z1} \left(\frac{p(\zeta)}{P_{z,z}} \right)^{-\varphi^z} \left(\frac{P_{z,z}}{P_z} \right)^{-\eta^z} (C_z + I_z) \\ & + n_{z2} \left(\frac{p_{z'}(\zeta)}{P_{z',z}} \right)^{-\varphi^{z'}} \left(\frac{P_{z',z}}{P_{z'}} \right)^{-\eta^{z'}} (C_{z'} + I_{z'}) \\ & + (1 - n_{z1} - n_{z2}) \left(\frac{p_{z''}(\zeta)}{P_{z'',z}} \right)^{-\varphi^{z''}} \left(\frac{P_{z'',z}}{P_{z''}} \right)^{-\eta^{z''}} (C_{z''} + I_{z''}), \end{aligned} \tag{53}$$

[30] This assumption can be easily relaxed assuming that a fraction of exporters invoice in terms of the foreign currency. Adjemian *et al.* (2008) estimate the share of firms that follow local currency pricing (LCP) or producer-currency-pricing (PCP) strategies. Their evidence for the Euro area and the U.S. seems to favor the PCP strategy.

where in the RHS we can identify the domestic demand and two foreign demands (exports) with prices given by $p_{z'}(\zeta) = p(\zeta)/S_{z,z'}$ and $p_{z''}(\zeta) = p(\zeta)/S_{z,z''}$. In addition, we employed per capita consumption aggregates definitions: $C_z \equiv \frac{1}{n_z}\int_0^{n_z} C^{j^z} dj^z$, $C_{z'} \equiv \frac{1}{n_{z'}}\int_0^{n_{z'}} C^{j^{z'}} dj^{z'}$ and $C_{z''} \equiv \frac{1}{1-n_z-n_{z'}}\int_{n_z+n_{z'}}^1 C^{j^{z''}} dj^{z''}$ and similarly for investment. Then, a formal expression for $Y^d(\zeta)$ reads as:

$$Y_t^d(\zeta) = \int_0^1 \left[C_t^j(\zeta) + I_t^j(\zeta)\right] dj + g_t(\zeta) + K_{z,t-1}\Phi\left(u_{z,t}\right), \tag{54}$$

where $g_t(\zeta)$ comes from Equation (35), while resources lost by adjusting the capital utilization must take into account the integration over individuals, $K_{z,t-1}\Phi\left(u_{z,t}\right) = K_{z,t-1}\int_0^{n_z}\Phi\left(u_t^{j^z}\right) dj^z$.

In short, private consumption plus investment from Equation (53), $G_{z,t}$ and $K_{z,t-1}\Phi\left(u_{z,t}\right)$ combined bring about the worldwide per capita demand for the variety produced at home, ζ.[31]. Similarly, in the following section we arrive at aggregates by integrating over varieties ζ.

Integration among varieties ζ Notice that the procedure is not as straightforward as in the previous section because there is price dispersion at the variety level that has to be taken into account. Following Schmitt-Grohe & Uribe (2005) and to keeps things simple, we assume identical price dispersions of any firm that belongs to country z no matter if it exports or if it serves only the domestic market.[32] Formally, price dispersion at time t is defined as:[33]

$$\Delta_z \equiv \left(\frac{1}{n_{z1}}\right)\int_0^{n_z}\left(\frac{p(\zeta)}{P_{z,z}}\right)^{-\varphi^z} d\zeta. \tag{55}$$

Price dispersion brings about inefficiencies in household consumption decisions because while prices differ, firms' marginal cost is identical (given our assumptions on technology). Consequently, price dispersion will not necessarily signal consumers to an optimal allocation, i.e., where consumption expenses to buy a determined bundle are minimized.

To determine, for instance, the exact form of (55), notice that price dispersions are linked to the operative pricing rule. Hence, expansion of Equation (55) under the assumption of Calvo price setting yields:

$$\begin{aligned}\Delta_{z,t} &= \left(P_{z,z,t}\right)^{\varphi^z}\left(\frac{1}{n_{z1}}\right)\int_0^{n_{z1}}\left(p_t(\zeta)\right)^{-\varphi^z} d\zeta \\ &= \left(P_{z,z,t}\right)^{\varphi^z}\left(\frac{1}{n_{z1}}\right)\int_0^{n_{z1}}\left[\sum_{h=0}^{\infty}\left(1-\alpha_P^z\right)\left(\alpha_P^z\right)^h\left(\tilde{p}_{t-h}(\zeta)\right)^{-\varphi^z}\right] d\zeta,\end{aligned}$$

where $\left(1-\alpha_P^z\right)\left(\alpha_P^z\right)^h$ is the share of resetting firms at time h, while $\tilde{p}_{t-h}(\zeta)$ comes from (48) after assuming symmetry. Extracting $\left(1-\alpha_P^z\right)$ from the summation and expanding it backwardly, we obtain an expression that is substituted into Equation (51). Hence, $\Delta_{z,t}$ can be written in a recursive form:[34]

$$\Delta_{z,t} = P_{z,z,t}^{\varphi^z}\left(1-\alpha_P^z\right)\tilde{p}_t(\zeta)^{-\varphi^z} + \alpha_P^z\left(\frac{P_{z,z,t}}{P_{z,z,t-1}}\right)^{\varphi^z}\Delta_{z,t-1}, \tag{56}$$

[31] If $\mu_{P,t} = \mu_{P,t}^z$ and $\eta = \eta^z(\forall z \in Z)$, Equation (54) can be rewritten as follows: $Y^d(\zeta) = \left(\frac{p(\zeta)}{P_{z,z}}\right)^{-\varphi^z}\left[\left(\frac{P_{z,z}}{P_z}\right)^{-\eta}\left[C_W(\zeta) + I_W(\zeta)\right] + G_z\right] + K_{t-1}\Phi\left(u_t\right)$, where we used definitions $C_W(\zeta) \equiv \int_0^1 C^j(\zeta)dj$ and $I_W(\zeta) \equiv \int_0^1 I^j(\zeta)dj$.

[32] Adjemian *et al.* (2008) distinguish among the market served for a certain domestic producer i^z, meaning that price dispersion of varieties sold domestically may differ from price dispersions of exported varieties. That distinction in the modeling allows them to study LCP.

[33] A similar expression is found in Di-Giorgio & Nistico (2007).

[34] As suggested by Woodford (2003) on page 399.

where $\tilde{p}_t(\zeta)$ is given by Equation (48). If prices were flexible ($\alpha_P^z = 0$), then $\tilde{p}_t(\zeta) = P_{z,z,t}$ for all ζ, thus $\Delta_{z,t} = 1$ and we conclude that the efficiency condition holds. Under sticky prices, $\Delta_{z,t} > 1$, thus consumption decisions will be distorted. Likewise, we may get $\Delta_{z',t}$ and $\Delta_{z'',t}$.

Taking into account these price dispersion definitions, the RHS of Equation (52) can be expanded in a consistent way. We concentrate on private and public demand. First, at period t, per capita private demand (home and foreign imports) is given by Equation (53), which integrated over varieties ζ yields:

$$\frac{C_z + I_z}{\Delta_z} = n_{z1}\left(\frac{P_{z,z}}{P_z}\right)^{-\eta^z}(C_z + I_z) + n_{z2}\left(\frac{P_{z',z}}{P_{z'}}\right)^{-\eta^{z'}}(C_{z'} + I_{z'})$$
$$+ (1 - n_{z1} - n_{z2})\left(\frac{P_{z'',z}}{P_{z''}}\right)^{-\eta^{z''}}(C_{z''} + I_{z''})\,. \tag{57}$$

Second, government's aggregate demand (35) reads as $G_z(\zeta) = \int_0^{n_{z1}}\left(\frac{p(\zeta)}{P_z}\right)^{-\varphi^z} G_z d\zeta$, which taking into account Equation (55) can be rewritten as:

$$G_z(\zeta) = \Delta_z\left(\frac{P_{z,z}}{P_z}\right)^{\varphi^z} G_z, \tag{58}$$

where G_z is an exogenous process defined in (36).

3.4.2 Aggregate supply and market clearing condition

To derive consistently country z's output we simply need to integrate the production function (40) across firms:

$$Y_{z,t} \equiv \int_0^{n_z} Y_t^{i^z} di^z = \int_0^{n_z}\left[A_{z,t}\left(K_{t-1}^{i^z}\right)^{\alpha^z}\left(L_t^{i^z}\right)^{1-\alpha^z} - FC_z\right] di^z = A_{z,t}\left(K_{z,t-1}\right)^{\alpha^z}\left(L_{z,t}\right)^{1-\alpha^z} - FC_z.$$

In equilibrium, the *economy*'s output, $Y_{z,t}$, equals the aggregate demand, Y_z^d, presented in the RHS of Equation (52):

$$\frac{Y_z}{\Delta_{z,t}} = \frac{n_{z1}}{n_z}\left(\frac{P_{z,z}}{P_z}\right)^{-\eta^z}(C_{z,t} + I_{z,t}) + \frac{n_{z2}}{n_{z'}}\left(\frac{P_{z',z}}{P_{z'}}\right)^{-\eta^{z'}}(C_{z',t} + I_{z',t})$$
$$+ \frac{(1 - n_{z1} - n_{z2})}{1 - n_z - n_{z'}}\left(\frac{P_{z'',z}}{P_{z''}}\right)^{-\eta^{z''}}(C_{z'',t} + I_{z'',t}) + \left(\frac{P_{z,z}}{P_z}\right)^{\frac{\mu_{P,t}^z}{\mu_{P,t}^z - 1}} G_{z,t} + \frac{\Phi\left(u_{z,t}\right) K_{z,t-1}}{\Delta_{z,t}}. \tag{59}$$

3.4.3 Assets and inputs markets equilibrium

Net assets holdings from the point of view of country z is defined as $\mathcal{F}_{z,t} \equiv \sum_{z' \in Z^c}(S_{z,z',t}B_{z',z,t} - S_{z',z,t}B_{z,z',t})$, where $S_{z,z',t} = S_{z',z,t}^{-1}$ by symmetry. It follows that in equilibrium (See, Adjemian *et al.* (2008)):

$$\mathcal{F}_{z,t} + \mathcal{F}_{z',t} + \mathcal{F}_{z'',t} = 0.$$

The capital rental market is in equilibrium when the demand for capital by goods producers equals the agents' supply at the price that clears the market, $R_{z,t}^k$.[35] As a result, the whole-economy utilization rate $u_{z,t}$ is determined as well. Moreover, the labor market is in equilibrium if firms'

[35] Note that if we would consider the case of capital treated as firm specific, there would be no supply of capital; hence, it does not make sense to think of an equilibrium rental rate of capital.

demand for labor equals labor supply at the wage level, $W_{z,t}$, given by Equation (3). The interest rate is determined by a Taylor rule that summarizes MP decisions (see Section 5.1). In the capital market, equilibrium means that the government debt is held by domestic and foreign investors at the market interest rate $r_{z,t} \equiv R_{z,t} - 1$.

4 Welfare metric

In general, under certain conditions, it is possible to define an aggregate welfare metric for country z by integrating over agents j^zs' utilities:[36]

$$\mathsf{We}_{z,t} \equiv E_t \left[\sum_{h=0}^{\infty} \beta^h \left(\left(\frac{1}{n_z} \right) \int_0^{n_z} U_{t+h}^{j^z} dj^z \right) \right], \tag{60}$$

where $U_{t+h}^{j^z}$ is defined in Equation (4). Thus, we can rewrite (60) —assuming that $\sigma_C^z \to 1$— as:

$$\mathsf{We}_{z,t} = E_t \sum_{h=0}^{\infty} \beta^h \left[\varepsilon_{U,t+h}^z \ln \left(C_{z,t+h} - h^z C_{z,t+h-1} \right) - \frac{\varkappa_L^z \varepsilon_{L,t+h}^z}{1+\sigma_L^z} \tilde{\Delta}_{W,z,t+h} \right], \tag{61}$$

where we assumed that the agent j^z's consumption equalizes consumption of all other agents, $\int_0^{n_z} C^{j^z} dj^z = C_z$ and additionally, we substituted $\left(\frac{1}{n_z}\right) \int_0^{n_z} \left(L_t^{j^z}\right)^{1+\sigma_L^z} dj^z$ by $\tilde{\Delta}_{W,z,t}$, which taking into account (2) expands to:

$$\tilde{\Delta}_{W,z,t} = L_{z,t}^{1+\sigma_L^z} \left(\frac{1}{n_z} \right) \int_0^{n_z} \left(\frac{W_t^{j^z}}{W_{z,t}} \right)^{-\sigma_W^z \left(1+\sigma_L^z\right)} dj^z. \tag{62}$$

Proceeding similarly as in Section 3.4.1, in Appendix A.4 we show that wage dispersion has a recursive form inherited from Equation (18) that reads as:

$$\Delta_{W,z,t} = \left(\frac{\tilde{w}_{z,t}}{w_{z,t}} \right)^{-\sigma_W^z \left(1+\sigma_L^z\right)} + \alpha_W^z \left(\frac{w_{z,t}}{w_{z,t-1}} \right)^{\sigma_W^z \left(1+\sigma_L^z\right)} \Pi_{z,t}^{\sigma_W^z \left(1+\sigma_L^z\right)} \Delta_{W,z,t-1}, \tag{63}$$

where $\Pi_{z,t} \equiv \frac{P_{z,t}}{P_{z,t-1}}$, $w_{z,t} \equiv \frac{W_{z,t}}{P_{z,t}}$, $\tilde{w}_{z,t} \equiv \frac{\tilde{W}_{z,t}}{P_{z,t}}$, wage dispersion is defined as $\Delta_{W,z,t-1} \equiv \sum_{h=0}^{\infty} \left(\alpha_W^z\right)^h \left(\frac{\tilde{W}_{z,t-h-1}}{W_{z,t-1}} \right)^{-\sigma_W^z}$ and $\tilde{\Delta}_{W,z,t} \equiv \left(1-\alpha_W^z\right) L_{z,t}^{1+\sigma_L^z} \Delta_{W,z,t}$.[37]

The intuition dictates that if wages are flexible, i.e., $\alpha_W^z = 0$, then $\tilde{W}_{z,t} = W_{z,t}$ and firms hire the same amount of work from each agent, leading to no dispersion. Therefore, Equation (63) simplifies to $\Delta_{W,z,t} = 1$ and $\tilde{\Delta}_{W,z,t} = L_t^{1+\sigma_L^z}$. Given that agents consume the same and supply the same hours of work, it follows that welfare equalizes individual utility: $\mathsf{We}_{z,t} = U_{t+h}^{j^z}$. On the other hand, if wages are sticky ($\alpha_W^z > 0$), there is a dispersion of wages that makes firms hire different amounts of work from each agent, but since workers identical and have a similar MRS, it follows that this inefficiency gives rise to a lower welfare level than when $\alpha_W^z = 0$ (the competitive labor market equilibrium).

[36] Likewise, foreign economies' welfare metrics are $\mathsf{We}_{z',t} \equiv E_t \left[\sum_{h=0}^{\infty} \beta^h \left(\left(\frac{1}{n_{z'}} \right) \int_0^{n_{z'}} U_{t+h}^{j^{z'}} dj^{z'} \right) \right]$ and $\mathsf{We}_{z'',t} \equiv E_t \left[\sum_{h=0}^{\infty} \beta^h \left(\left(\frac{1}{1-n_z-n_{z'}} \right) \int_{n_z+n_{z'}}^{1} U_{t+h}^{j^{z''}} dj^{z''} \right) \right]$.

[37] The wage dispersion employed in the software differs from (63): $\Delta_{W,z,t} = \left(\breve{w}_{z,t}\right)^{-\sigma_W^z \left(1+\sigma_L^z\right)} + \alpha_W^z \left(\frac{W_{z,t-1}}{W_{z,t}} \right)^{-\sigma_W^z \left(1+\sigma_L^z\right)} \Delta_{W,z,t-1}$, where $\breve{w}_{z,t}$ is defined as $\tilde{W}_{z,t}/W_{z,t}$. As $\tilde{\Delta}_{W,z,t}$ enters into (61), we must not confuse it with $\Delta_{W,z,t}$.

However, it is difficult to compare the levels of utility derived under different degrees of rigidities or alternative policies. Lucas (1987) and more recently Lucas (2003) pioneered in the quantitative evaluation of welfare costs in terms of steady-state consumption. Given a particular monetary regime, we measure business fluctuations welfare costs as the permanent shift in the steady-state consumption that would deliver identical utility if the fluctuations were eliminated. In other words, it is interpreted as the consumption that would be voluntarily given up by consumers to assure hypothetical paths of stochastic processes that do not cause utility fluctuations. Following Bergin & Tchakarov (2003), we may compute the (conditional) individual welfare cost from business cycles by μ^{j^z}, such that:[38]

$$\begin{aligned}\breve{U}_t^{j^z}\left[\left(1+\mu^{j^z}\right)C_z, L_z\right] &= E_t\left\{\breve{U}_t^{j^z}\left[\left\{\varepsilon^z_{U,t+h}\right\}_0^\infty, \left\{C^{j^z}_{t+h}\right\}_0^\infty, \left\{\varepsilon^z_{L,t+h}\right\}_0^\infty, \left\{L^{j^z}_{t+h}\right\}_0^\infty\right]\right\}, \\ &= (1-\beta)\sum_{h=0}^{\infty}\beta^{t+h}E_0\left[U^{j^z}_{t+h}\right].\end{aligned}$$

At the aggregate level, we seek to calculate μ^z by employing Equation (61) in the previous formulation, so we obtain:

$$\ln\left[\left(1-h^z\right)\left(1+\mu^z\right)C_z\right] - \frac{\varkappa^z_L}{1+\sigma^z_L}\tilde{\Delta}_{W,z}^{1+\sigma^z_L} = \left\{(1-\beta)\sum_{h=0}^{\infty}\beta^{t+h}E_0\left[\mathsf{We}_{z,t+h}\right]\right\},$$

from where it is straightforward to isolate μ^z:

$$\mu^z = \left[\left(1-h^z\right)C_z\right]^{-1}\left\{\exp\left\{\left((1-\beta)\sum_{h=0}^{\infty}\beta^{t+h}E_0\left[\mathsf{We}_{z,t+h}\right]\right) + \frac{\varkappa^z_L}{1+\sigma^z_L}\tilde{\Delta}_{W,z}^{1+\sigma^z_L}\right\}\right\} - 1. \tag{64}$$

5 Monetary and Fiscal policy

In this study a key maintained assumption is that both the fiscal authority and the central bank are committed to previously announced rules that make their behavior expected and anticipated by the all agents. In the following subsections we analyze monetary and fiscal policy.

5.1 Monetary policy

Beginning with MP we assume that the home and foreign countries have independent CBs. These monetary authorities manage MP instruments such as the short-run interest rate. As Taylor (1993) have shown, a simple and parsimonious interest rate rule that targets inflation and the trend GDP, could capture the behavior of the FED surprisingly well. The so-called Taylor rule is assumed as given (that means that the design of MP was done in an early stage, and from that point in time on, the Taylor rule has been fully operative and agents expect that it will maintain in the near future). For the sake of concreteness the non-linear Taylor rule is specified as:

$$\frac{1+r_{z,t}}{1+r_z} = \left(\frac{1+r_{z,t-1}}{1+r_z}\right)^{\rho^z_R}\left[\left(\frac{1+\pi_{z,t}}{1+\pi_z}\right)^{\phi^z_\pi}\left(\frac{gdp_{z,t}}{gdp_z}\right)^{\phi^z_Y}\right]^{1-\rho^z_R} e^{\varepsilon^z_{R,t}}, \left\{\begin{array}{ll}\text{pre-EMU,} & \forall z\in Z\\ \text{EMU, for } z=\{EA,US\}\end{array}\right., \tag{65}$$

[38] There are two approaches to calculate welfare, conditional —as followed here— and unconditional. That choice is justified because the aim of this paper does not consider the issue of policy inconsistency, neither the commitment vs. discretion dilemma. See Bergin & Tchakarov (2003) and Kollmann (2002).

where $r_{z,t}$ is the operative instrument of the CB that depends on the lagged nominal interest rate, $r_{z,t-1}$, CPI inflation rates, $\pi_{z,t}$, real output $gdp_{z,t}$. In the EMU regime the OeNB does not manage MP but the ECB does, so that $r_{AT,t} = r_{EA,t}$.

Notice that all variables are measured in terms of deviations from their corresponding steady state values (these omit time sub-indices). Finally, $\varepsilon^z_{R,t}$ is a zero-mean innovation with constant variance $\sigma^2_{\varepsilon^z_R}$, that can be interpreted as an unexpected money demand shock.[39] The elasticity ρ^z_R measures the degree of concern of the CB in managing the instrument in a smooth way. The lower ρ^z_R the more concern is placed on the actual output gap and current CPI inflation fluctuations to determine the nominal interest rate. Notice that if $\phi^z_Y = 0$, the rule is known as inflation targeting.

5.2 Fiscal policy

This section begins providing reasons for a plausible modeling of fiscal policy for Austria, the EA and the U.S.

Firstly, Austria and the EA are obliged to follow the budgetary rules of the SGP since the start of EMU in 1999. A reform of the Pact took place in 2005 (See, Breuss (2007)). In short, this implies a balanced budget over the business cycle and that the deficit must not exceed 3% of GDP in one particular year

Secondly, in 1985 the U.S. federal government introduced deficit controls through the Gramm-Rudman-Hollings (GRH) law, also known as the Balanced Budget and Emergency Deficit Control Act. It meant a convergence to a zero deficit in a horizon of six years, then extended to eight years. Leaving aside Social Security, the effort materialized mainly through spending cuts. A new legislation complemented the GRH law to fix some holes, the Budget Enforcement Act of 1990 (BEA) shifted the focus away from deficit targets toward expenditure and revenue controls.[40] Its application was effective till 1998 (when zero deficit was reached) and further extended to 2002, but in this latter period interval it was not truly operative. Overall, there is a consensus on that this legislation improved fiscal discipline in the U.S..

In summary, any fiscal rule had as a common goal to put in practice —in a enforced way—, the principle of healthy public finances that points out that it would be desirable to use debt finance over the business cycle has been followed in several instances.[41]

Considering all aspects just mentioned, we assume that the transfer's scheme will be the most flexible tool to react to business cycle fluctuations. In particular, we will assume that transfers (or lump sum taxes if negative) are set by the government according to a zero deficit rule:

$$T_{z,t} = TX_{z,t} - P_{z,t}G_{z,t} - (\mathcal{S}^z_P - 1)\int_0^{n_z} \tilde{p}^{i^z}_t(\zeta)Y^{i^z}_{z,t+h}di^z - (\mathcal{S}^z_W - 1)\int_0^{n_z} W^{j^z}_t L^{j^z}_t dj^z,$$

which comes from the period GBC, Equation (38), under assets market equilibrium.[42]

6 Exogenous shocks and normalization

In this section we focus on two issues. First, we present the structure of exogenous variables and shocks included in the model. Second, we normalize all nominal variables with the relevant

[39] Notice that $\pi_{z,t}$ is inflation, whereas gross CPI inflation is defined as $\Pi_{z,t} \equiv P_{z,t}/P_{z,t-1}$.

[40] It was hard to control deficit reductions ex ante employing the GRH law, because targets were applied to the projected, rather than to actual deficits.

[41] Indeed, the United Kingdom (U.K.) explicitly adopted this principle into the so-called golden rule.

[42] In Section 3.2 we described the different tax rates applied; however, nothing was said about the fact that these are fixed parameters. Forni *et al.* (2007), for instance, allow for endogenous business cycle fluctuations of tax rates and find a great deal of persistency on them; however, we follow the specification suggested by Andres *et al.* (2006) because, under some circumstances, these fluctuations could lead to indeterminacy. Moreover, to change the tax structure often involves administrative costs and delays that need to be taken into account. Thus, we would like to stress the permanent character of the rule and higher costs to modify it.

aggregate price index.

Beginning with the structure of exogenous variables, the relevant vector for country z is defined as $\xi_{z,t} \equiv (\ln A_{z,t},\ \ln g_{z,t},\ \ln \mu^z_{P,t},\ \ln \mu^z_{W,t})'$ with mean $\xi_z = (\ln A_z,\ \ln g_z,\ \ln \mu^z_P,\ \ln \mu^z_W)'$, that implies the world economy involves the concatenation, $\xi_t \equiv (\xi_{z,t},\ \xi_{z',t},\ \xi_{z'',t})'$. These exogenous processes follow a VAR(1) specification:

$$\xi_t = \rho_\xi \xi_{t-1} + \varepsilon^0_t, \tag{66}$$

where ε^0_t is a partition of ε_t (see below), and $\rho_\xi \equiv diag(\rho^z_A,\ \rho^z_G,\ \rho^z_{\mu_P},\ \rho^z_{\mu_W})$.[43]

Consider the structure of exogenous shocks defined by the relevant vector for country z, $\varepsilon_{z,t} \equiv (\varepsilon^z_{L,t},\ \varepsilon^z_{U,t},\ \varepsilon^z_{I,t},\ \varepsilon^z_{Y,t},\ \varepsilon^z_{G,t},\ \varepsilon^z_{R,t},\ \varepsilon^z_{\mu_P,t},\ \varepsilon^z_{\mu_W,t})'$ with mean $(1,\ 1,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0)'$. We define $\varepsilon^0_t \equiv (\varepsilon^z_{Y,t},\ \varepsilon^z_{G,t},\ \varepsilon^z_{\mu_P,t},\ \varepsilon^z_{\mu_W,t})'$, which are the disturbances of exogenous processes, but notice that $\varepsilon^z_{\mu_P,t} \equiv \nu^z_{P,t} - \vartheta^z_P \nu^z_{P,t-1}$ and $\varepsilon^z_{\mu_W,t} \equiv \nu^z_{W,t} - \vartheta^z_W \nu^z_{W,t-1}$ to abbreviate notation. Likewise, the world economy shocks vector is the concatenation, $\varepsilon_t \equiv \begin{pmatrix} \varepsilon_{z,t} & \varepsilon_{z',t} & \varepsilon_{z'',t} \end{pmatrix}'$. To account for both temporary and permanent effects of shocks, we assume the following VAR(1) specification:

$$\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \nu_t, \tag{67}$$

where $\rho^z_\varepsilon \equiv diag(\rho^z_L,\ \rho^z_U,\ \rho^z_I,\ 0,\ 0,\ 0,\ 0,\ 0)$ and $\rho_\varepsilon \equiv \begin{pmatrix} \rho^z_\varepsilon & \rho^{z'}_\varepsilon & \rho^{z''}_\varepsilon \end{pmatrix}'$. Moreover, $\nu_t \equiv (\nu^z_{L,t},\ \nu^z_{U,t},\ \nu^z_{I,t},\ \nu^z_{A,t},\ \nu^z_{G,t},\ 0,\ 0,\ 0)'$ has mean the zero vector and variance-covariance matrix $\mathbf{\Sigma}_{z,\nu} = diag(\sigma^2_{\nu^z_L},\ \sigma^2_{\nu^z_U},\ \sigma^2_{\nu^z_I},\ \sigma^2_{\nu^z_A},\ \sigma^2_{\nu^z_G},\ 0,\ 0,\ 0)$ and $\mathbf{\Sigma}_\nu \equiv diag\left(\mathbf{\Sigma}_{z,\nu},\ \mathbf{\Sigma}_{z',\nu},\ \mathbf{\Sigma}_{z'',\nu}\right)$.

The normalization assures stationary of all model variables. Recall that two key aspects may cause nonstationarity of variables. First, nominal variables (e.g., prices) are clearly non-stationary, so we must find a way to rewrite them as relative prices. Second, if the technology process has a unit root, we need to transform real variables in a way that include the technological progress that drives them. However, in view of Equation (41), the latter aspect will not apply for our model.

The normalization assures that the steady state is well defined, and is consistent with the fixed point definition. Thus we deflate all nominal variables in the model by the whole economy price level, P_z. To keep notation as simple as possible, we omit time subscripts (unless needed). That means that all real variables in the model remain the same, while the following ones need to be defined: $m^{j^z} \equiv \frac{M^{j^z}}{P_z}$, $w^{j^z} \equiv \frac{W^{j^z}}{P_z}$, $r^k \equiv \frac{R^k}{P_z}$, $mc_z \equiv \frac{MC_z}{P_z}$, $p_{z,z} \equiv \frac{P_{z,z}}{P_z}(z \in Z)$, $p_{z,z'} \equiv \frac{P_{z,z'}}{P_z}$ and $p_{z',z} \equiv \frac{P_{z',z}}{P_{z'}}(z \in Z$ and $z' \in Z^c)$. Optimal prices set by following the Calvo pricing rule as well as wages must also be deflated. Home producers will charge $\widetilde{rp}_z(\zeta) \equiv \frac{\tilde{p}_z(\zeta)}{P_z}(z \in Z)$, $\mathcal{RP}_{i,z} \equiv \frac{\mathcal{P}_{1,z}}{P_z}(i = 1, 2;\ z \in Z)$, while real wages are $\widetilde{rw}_z \equiv \frac{\tilde{W}_z}{P_z}$, $\mathcal{RW}_{i,z} \equiv \frac{\mathcal{W}_{i,z}}{P_z}$ $(i = 1, 2;\ z \in Z)$. Moreover, real profits are defined as $\mathsf{RPr}_z \equiv \frac{\mathsf{Pr}_z}{P_z}$. Furthermore, we define the nominal exchange rate depreciation rate $\Delta S_{z,z',t} \equiv \frac{S_{z,z',t}}{S_{z,z',t-1}}$ $(z \in Z$ and $z' \in Z^c)$. Finally, an additional transformation is made to the newly set (Calvo) wage, $\breve{w}_{z,t} \equiv \frac{\tilde{W}_{z,t}}{W_{z,t}}$, $\breve{\mathcal{W}}^1_{z,t} \equiv \mathcal{W}^1_{z,t} W_{z,t}^{-\sigma^z_W\left(1+\sigma^z_L\right)}$ and $\breve{\mathcal{W}}^2_{z,t} \equiv \mathcal{W}^2_{z,t} W_{z,t}^{1-\sigma^z_W}$, so that Equation (17) is written as:

$$\left(\breve{w}_t\right)^{1+\sigma^z_L \sigma^z_W} = \frac{\mu^z_{W,t}}{\left(1-\tau^z_w\right)\mathcal{S}^z_W} \varkappa^z_L \frac{\breve{\mathcal{W}}^1_{z,t}}{\breve{\mathcal{W}}^2_{z,t}}, \quad \text{where} \tag{68}$$

[43] Notice that the variance of ε is $\mathbf{\Sigma}_{z,\varepsilon} = diag(\sigma^2_{\varepsilon^z_L},\ \sigma^2_{\varepsilon^z_U},\ \sigma^2_{\varepsilon^z_I},\ \sigma^2_{\varepsilon^z_A},\ \sigma^2_{\varepsilon^z_G},\ \sigma^2_{\varepsilon^z_R},\ \sigma^2_{\varepsilon_{\mu^z_P}},\ \sigma^2_{\varepsilon_{\mu^z_W}})$. However, in the estimation results we report $\sigma^2_{\mu^z_P}$ and $\sigma^2_{\mu^z_P}$ instead

$$\check{\mathcal{W}}^1_{z,t} \equiv \varepsilon^z_{L,t} \left(L_{z,t}\right)^{1+\sigma^z_L} + \beta^z \alpha^z_W E_t \left[\Pi^{\sigma^z_W\left(1+\sigma^z_L\right)}_{W,z,t+1} \check{\mathcal{W}}^1_{z,t+1} \right],$$
$$\check{\mathcal{W}}^2_{z,t} \equiv \Lambda_{z,t} L_{z,t} w_{z,t} + \beta^z \alpha^z_W E_t \left[\Pi^{\sigma^z_W - 1}_{W,z,t+1} \mathcal{W}^2_{z,t+1} \right].$$

Accordingly, Equation (63) is replaced by:

$$1 = \alpha^z_W \Pi^{\frac{1}{\mu^z_{W,t}-1}}_{W,z,t} + \left(1 - \alpha^z_W\right) \left(\check{w}_{z,t}\right)^{-\frac{1}{\mu^z_{W,t}-1}}, \tag{69}$$

and to pin down the wage inflation we add the following identity:

$$\frac{w_{z,t}}{w_{z,t-1}} \equiv \frac{\Pi_{W,z,t}}{\Pi_{z,t}}. \tag{70}$$

Employing these variable definitions to substitute for all nominal variables, we are able to work with a more reduced system comprising only real variables. This is nothing more than taking the whole economy aggregate price as numéraire. These modified relationships included into the model would not have any implication in terms of the equilibrium allocation, namely, the steady state which is fully consistent in terms of real variables.

Conditional on the parameterization employed (see below), the modified nonlinear model equations are arranged in an implicit function with matrices as arguments:[44]

$$E_t\{f(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{v}_t; \theta)\} = \mathbf{0}, \tag{71}$$

where calibrated parameters are gathered in the vector θ, $\mathbf{v}_t$ are structural shocks that drive exogenous processes explained in Equation (67) and $\mathbf{y}_t$ represents all endogenous variables of the model.

First the model is solved at the non-stochastic steady state, so that Equation (71) at the steady state becomes: $E_t\{f(\mathbf{y}, \mathbf{y}, \mathbf{y}, \mathbf{0}; \theta)\} = \mathbf{0}$. The nonlinear DYNARE solver running MATLAB calculates the steady state.

7 Data and measurement assumptions

Data availability is quite suitable to estimate our DSGE model; the U.S. series start from 1954:Q2 to 2008:Q3, whereas for the EA we make use of Fagan *et al.* (2001) dataset in its most recent update comprising EMU15 aggregates. Unfortunately, it is not available from the notes of the eight update, the specific weights assigned to each of the 15 countries. Weights are reproduced by Breuss & Rabitsch (2009) for the EU12

The working sample is conditioned by two facts that must be taken into account. First, following Smets & Wouters (2007), we intend to identify parameters of the MP rule that are consistent with the so-called Great Moderation (GM) for the U.S., meaning that the sample covers the period 1984:Q1-2007:Q4. Second, we must take into account the regime shift that the beginning of the EMU implied; therefore, we divide the period in the pre-EMU period (1984Q1-1998Q4) and the EMU period (1999Q1-2007Q4). As a result, we count with two samples of extension $T_1 = 60$ and $T_2 = 36$.

Observed variables that are nonstationary such as GDP, private consumption and investment are transformed in per-worker terms (proxied by the labor force). Further, these are logged and then detrended by employing the Hodrick-Prescott Filter with smoothing parameter $\lambda = 1600$.

[44] A transformation of variables is made $\nu_{P,t-1} = ma_{P,t}$ and $\nu_{W,t-1} = ma_{W,t}$ to make $ma_{P,t}$ and $ma_{W,t}$ endogenous and consequently eliminate $\mathbf{v}_{t-1}$ from the specification.

Regarding the measures of price and wage inflation, we calculate gross rates and detrend them by the same method. Finally, nominal interest rates are in percentage points and we do not apply any transformation on them.

Given these transformations, Table 1 illustrates stylized business cycle facts: relative standard deviations (SDs) of our variables w.r.t. to output's SD. The table is divided in three stacked panels, Austria is at the top, whereas the EA is in the middle and the U.S. is at the bottom part. Across columns Table 1 presents different subsamples. In general, it is observed that consumption is less volatile that output except for the U.S. in the period 1984Q1-1998Q4; however, in the whole sample period it is lower than one. As Breuss & Rabitsch (2009) point out, this is a characteristic of developed economies. Furthermore, investment is in all instances more volatile than output, indeed a notorious fact is that investment of the U.S. is more than seven times more volatile than output. For Austria and the EA investment volatility ranges from 1.6 to 2.7 depending on the subsample considered. Our results based on per-worker consumption aggregates contrast with results reported by Breuss & Rabitsch (2009). The EMU clearly lowered the volatility of consumption as well as of investment of the EA, results which are quite reasonable.

Following with price and wage inflation, we observe that the former remains higher that the other in Austria (the reverse is the case for the EA), whereas they are quite similar for the U.S.. Moreover, nominal interest rates tend to be more volatile that output, except in the EMU period for Austria and the EA. Finally, the gross depreciation rate of € w.r.t. the USD is much more volatile than the one for the Austrian Schilling (S) w.r.t. the €. This is not surprising in view of the early anchoring of the Austrian Schilling to the Deutsche Mark by mid 1976.

Table 1: Stylized business cycle facts from data

SDs of observed variables w.r.t. SD of observed Output

Country/Subsample	Entire period	pre-EMU	GM	GM, pre EMU	GM, EMU
Austria					
Output	1.000	1.000	1.000	1.000	1.000
Consumption	0.806	0.847	0.788	0.851	0.721
Investment	2.355	2.654	1.755	1.922	1.626
Price inflation	0.356	0.393	0.360	0.444	0.272
Wage inflation	0.227	0.264	0.224	0.299	0.126
Nom. int. rate	2.486	2.136	1.999	1.625	0.976
Gross Depr. Rate (S/€)	1.005	1.208	0.803	1.154	0.002
EA					
Output	1.000	1.000	1.000	1.000	1.000
Consumption	0.939	0.995	0.925	0.996	0.699
Investment	2.619	2.626	2.708	2.751	2.548
Price inflation	0.289	0.294	0.238	0.221	0.274
Wage inflation	0.470	0.505	0.426	0.468	0.327
Nom. int. rate	3.385	2.717	2.822	2.259	0.773
Gross Depr. Rate (€/USD)	4.963	5.074	5.312	5.677	4.616
U.S.					
Output	1.000	1.000	1.000	1.000	1.000
Consumption	0.951	0.982	0.992	1.113	0.848
Investment	7.961	8.071	7.933	8.198	7.669
Price inflation	0.338	0.359	0.305	0.334	0.272
Wage inflation	0.328	0.349	0.342	0.408	0.263
Nom. int. rate	2.542	2.458	1.389	1.057	1.293

Table 2 reports cross-country correlations of home and foreign aggregates (for example, the correlation between Austrian consumption and consumption in the EA, and so on). Again, considering the GM sample and two subsamples: pre-EMU and EMU, the evidence suggests a sizeable co-movement of Austrian and EA variables, especially output, investment and nominal interest rates. It is noteworthy to remark that the price (wage) inflation co-movement rose (drop) to 0.625 (0.018) in the EMU period. When comparing the co-movement of Austrian variables with the ones from U.S., we observe correlations close to zero (indicative of almost independent behavior) in the GM sample, except for price inflation.

Table 2: Cross-correlations of Austrian variables w.r.t. respective foreign ones

Subsample	GM		pre-EMU		EMU	
Foreign country	EA	U.S.	EA	U.S.	EA	U.S.
Variables						
Output	0.805	-0.025	0.865	-0.114	0.749	0.060
Consumption	0.522	-0.013	0.523	0.138	0.522	-0.285
Investment	0.767	-0.051	0.722	-0.526	0.876	0.525
Price inflation	0.439	0.256	0.346	0.226	0.625	0.317
Wage inflation	0.176	-0.129	0.207	-0.220	0.018	0.178
Nom. int. rate	0.940	0.422	0.914	0.028	1.000	0.458
Gross Depr. Rate (S/€ and €/USD)	-0.059		-0.068		-0.040	

8 Estimation

This section briefly provides some intuition on the estimation methodology employed. Then, we provide motivation for the calibrated parameters. It is followed by the specification of estimates prior densities. Finally, we present results from Bayesian estimation.

8.1 Methodology

The model (71) is log-linearized around the steady state (first order approximation, i.e., $\hat{\mathbf{y}}_t \equiv \mathbf{y}_t - \mathbf{y}^*(\theta)$) where θ is the parameter vector. Sims (2002) suggested that the model can be reshuffled in the following linear state-space system:

$$\mathbf{B}_1\hat{\mathbf{y}}_t + \mathbf{B}_2\hat{\mathbf{y}}_{t-1} + \mathbf{C}\mathbf{v}_t + \mathbf{D}\eta_t = \mathbf{0}, \tag{72}$$

and solved employing a guess policy function. Consequently, (72) is handled with Sims (2002)'s algorithm, which allows us to obtain the following solution's representation:

$$\hat{\mathbf{y}}_t = \Xi_0\hat{\mathbf{y}}_{t-1} + \Xi_1\mathbf{v}_t. \tag{73}$$

Given Equation (73), a direct estimation approach would maximize its likelihood function with respect to θ and $vech(\mathbf{\Sigma}_v)$; however, we must acknowledge that not all variables included into $\hat{\mathbf{y}}_t$ are observed, a large number of them are unobserved. Hence, $\hat{\mathbf{y}}_t \equiv (\hat{\mathbf{y}}_t^o, \hat{\mathbf{y}}_t^{uno})'$, where in our model $\hat{\mathbf{y}}_t^{o\prime}$ has dimension 18×1 (pre-EMU) or 17×1 (EMU), for each t . Then, the complete state-space representation adds to (73) the following measurement equation:

$$\hat{\mathbf{y}}_t^o = \Upsilon\hat{\mathbf{y}}_t + \epsilon_t, \tag{74}$$

where Υ is a $18(17) \times n$ binary matrix that selects the observed variables from $\hat{\mathbf{y}}_t$, ϵ_t is a measurement error that is assumed to be *iid* with mean zero vector and variance $\mathbf{\Sigma}_\epsilon$.

Denoting the sample as $\hat{Y}_T^o \equiv \{\hat{\mathbf{y}}_1^o, \hat{\mathbf{y}}_2^o, \hat{\mathbf{y}}_3^o, ..., \hat{\mathbf{y}}_T^o\}$, the density of $\hat{Y}_T^o$ conditional on the parameters (likelihood) can be written as:[45]

$$\begin{aligned}\mathcal{L}(\theta, vech(\mathbf{\Sigma}_v), vech(\mathbf{\Sigma}_\epsilon); \hat{Y}_T^o) &= p(\hat{Y}_T^o \mid \theta, vech(\mathbf{\Sigma}_v), vech(\mathbf{\Sigma}_\epsilon)), \\ &= p(\hat{\mathbf{y}}_0^o \mid \theta, vech(\mathbf{\Sigma}_v), vech(\mathbf{\Sigma}_\epsilon)) \\ &\times \textstyle\prod_{t=1}^T p(\hat{\mathbf{y}}_t^o \mid Y_{t-1}^o, \theta, vech(\mathbf{\Sigma}_v), vech(\mathbf{\Sigma}_\epsilon)), \end{aligned} \tag{75}$$

which includes a marginal density (involving the distribution of the initial condition) $p(\hat{\mathbf{y}}_0^o \mid \theta, vech(\mathbf{\Sigma}_v), vech(\mathbf{\Sigma}_\epsilon))$ and a conditional density. Given our linearized model (73), (74) and our definition of ϵ_t, it follows that $\hat{\mathbf{y}}_0^o \sim \mathbf{N}(E_\infty[\hat{\mathbf{y}}_t^o], V_\infty[\hat{\mathbf{y}}_t^o])$.[46] Concerning the second factor, the conditional density involves the evaluation of $\hat{\mathbf{y}}_t^o \mid Y_{t-1}^o$ which is not directly observable since $\hat{\mathbf{y}}_t^o$ depends on other unobserved endogenous variables given by the model; however, it is at hand the following identity:

$$\begin{aligned}p(\hat{\mathbf{y}}_t^o \mid Y_{t-1}^o, \theta, vech(\mathbf{\Sigma}_v), vech(\mathbf{\Sigma}_\epsilon)) \equiv &\int_\Lambda p(\hat{\mathbf{y}}_t^o \mid \hat{\mathbf{y}}_t, \theta, vech(\mathbf{\Sigma}_v), vech(\mathbf{\Sigma}_\epsilon)) \\ &\times p(\hat{\mathbf{y}}_t \mid Y_{t-1}^o, \theta, vech(\mathbf{\Sigma}_v), vech(\mathbf{\Sigma}_\epsilon)) d\hat{\mathbf{y}}_t,\end{aligned}$$

where the density of $\hat{\mathbf{y}}_t^o \mid Y_{t-1}^o$ depends on the mean of the density of $\hat{\mathbf{y}}_t^o \mid \hat{\mathbf{y}}_t$ where the relevant weight is the density of $\hat{\mathbf{y}}_t \mid Y_{t-1}^o$. The former density is directly given by the measurement Equation (74), while $\hat{\mathbf{y}}_t \mid Y_{t-1}^o$ is computed by the Kalman filter.

8.1.1 Bayesian estimation: the likelihood meets prior densities

Bayesian estimation and evaluation techniques have been particularly successful in estimation of not only small DSGE models but also medium to large-scale New Keynesian models. The estimation procedure combines a likelihood function (75) derived from our model with the specification of a prior distribution for $\theta \equiv (\theta, vech(\mathbf{\Sigma}_v), vech(\mathbf{\Sigma}_\epsilon))'$. As a result, the state-space representation can be translated to form the posterior distribution.

The idea behind the Bayesian principle is to look for a parameter vector which maximizes the posterior density, given the prior and the likelihood based on the data. Formally, the posterior density $p(\theta|\hat{\mathbf{y}}^o)$ is related to the prior and the likelihood as follows:

$$p(\theta|\hat{\mathbf{y}}^o) = \frac{p(\hat{\mathbf{y}}^o|\theta)\, p(\theta)}{p(\hat{\mathbf{y}}^o)} \propto p(\hat{\mathbf{y}}^o|\theta)\, p(\theta) = L(\theta|\hat{\mathbf{y}}^o)\, p(\theta)), \tag{76}$$

where $p(\theta)$ is the prior density of the parameter vector, $\mathcal{L}(\theta|\hat{\mathbf{y}}^o)$ is the likelihood of the data and $p(\hat{\mathbf{y}}^o) = \int_\Theta p(\hat{\mathbf{y}}^o|\theta)\, p(\theta)\, d\theta$ is the unconditional data density, which, since it does not depend on the parameter vector to be estimated, can be treated as a proportionality factor and accordingly can be disregarded in the estimation process. Assuming *iid* priors, the logarithm of the posterior is given by the sum of the log likelihood of the data and the sum of the logarithms of the prior distributions:

$$\ln(p(\theta|\hat{\mathbf{y}}^o)) = \ln(\mathcal{L}(\theta|\hat{\mathbf{y}}^o)) + \sum_{\iota=1}^N \ln(p(\theta_i)). \tag{77}$$

The latter term can be directly calculated from the specified prior distributions of the estimated parameters. For the computation of the log likelihood of the data the Kalman filter is applied to

[45] Recall from Section 7, that we employ two samples $T_1 + T_2 = T$.

[46] Construction of the likelihood for an AR(1) and AR(p) processes are derived in Hamilton (1994) Ch.5, Sections 2 and 3, respectively. In case $\hat{y}_0^o$ contains variables with unit roots, the initialization assumes an infinite $V_\infty[\hat{\mathbf{y}}_s^o]$, which is known as diffuse Kalman filter.

the DSGE model solution (the state-state representation) for the number of periods, T, provided by the data $\hat{\mathbf{y}}^o$.

The (multivariate) posterior distribution for our DSGE model would not exist in closed form; however, it can be approximated through a Gaussian density providing the sample size grows.[47] Following Tierney & Kadane (1986), the posterior is understood as a kernel of unknown form, $\mathcal{K}(\theta) \equiv \mathcal{K}(\theta, Y_T^o)$, given that (one of) its mode is assumed to be known, θ^*, taking logs and approximating the kernel using a 2^{nd} order Taylor expansion, yields:

$$\log \mathcal{K}(\theta) \approx \log \mathcal{K}(\theta^*) - \frac{1}{2}(\theta - \theta^*)' \left[H(\theta^*)\right]^{-1} (\theta - \theta^*),$$

where $H(\theta^*)$ is minus the inverse of the Hessian of the model evaluated at the posterior mode. Consequently, the Gaussian posterior would be:

$$p(\theta^*) \approx (2\pi)^{T/2} \left|H(\theta^*)\right|^{-1/2} \exp\left\{-\frac{1}{2}(\theta - \theta^*)' \left[H(\theta^*)\right]^{-1} (\theta - \theta^*)\right\},$$

which enables us to approximate posterior moments, as derived by Kass *et al.* (1989) and Tierney *et al.* (1989).

The whole point is that the asymptotic approximation ($T \to \infty$) makes sense if and only if the true posterior does not differ from the hypothesized Gaussian. More exact results for our sample range can be derived via simulation given its non-standard shape, employing an approximation method around the optimum that generates a (large) sample of draws using the Markov-Chain Monte Carlo (MCMC) algorithm. This is useful to characterize the shape of the posterior distribution, from which inference can be drawn. The Metropolis-Hastings algorithm is implemented using a jumping distribution to visit areas that are not at the tails of the posterior. The validity of the "jump" is assessed via acceptance-rejection instrumented with the Metropolis-Hastings algorithm, where proposal draws that are accepted (rejected) are included (excluded) in Markov chain. The researcher establishes the ratio of acceptance. The simulation is considered large enough when pooled moments converge to within moments of the chain, see Brooks (1998).

8.2 Calibrated parameters and prior densities

The understanding that our model replicate processes over the business cycle frequency forces us to keep some parameters calibrated because data is uninformative on them. We assume that the (quarterly) willingness to wait, measured by β, is equal to $1.04^{-0.25}$ and the quarterly depreciation rate of physical capital, δ^z, is calibrated to 0.025 (which is equivalent to 10 per cent per year). Further, the shift parameter in the capital utilization cost function is assumed to be $\theta^z = 1$. Moreover, parameters of the utility function are calibrated as follows: $\sigma_C^z = 1$ so that consumption enters in a logarithmic way, the reverse of the labor Frisch elasticity, σ_L^z, is assumed equal to 6 and scale parameters $\varkappa_M^z = \varkappa_L^z = 1$ as in Canzoneri *et al.* (2007).[48] In addition, constants in real exchange rates are assumed $\kappa_1^z = \kappa_2^z = 1$, as in the literature, e.g., Adjemian *et al.* (2008).

We assume that the steady-state technology level, $\ln A_z$, is equal to 1.8. Moreover, price and wage markups, $\frac{\varphi^z}{\varphi^z - 1}$ and $\frac{\sigma_W^z}{\sigma_W^z - 1}$, are set to 1.2, consistent with substitution elasticities equal to 6. What is more, moving average parameters of the price and wage markups, ϑ_P^z and ϑ_W^z, are equal to 0.5. In addition, fixed cost of operating the firm is fixed and equal to 0.06 following Schmitt-Grohe & Uribe (2005). The fixed cost diminishes steady-state benefits to more reasonable values. $\rho_{\mu_P}^z$ and $\rho_{\mu_W}^z$ are calibrated to 0.95 as in Smets & Wouters (2007). In this version we shut down

[47] As the sample enlarges, the choice of the prior density would not affect the posterior.

[48] Galí (2008) in page 20 discusses the implications of $\sigma_C^z = 1$ (no unemployment). This assumption allows us to present results on welfare costs that are fully comparable in terms of steady state consumption units.

completely fiscal policy, allowing for a zero deficit rule with taxes that are lump sum (tax rates are all equalized to zero). Government spending relative to GDP amounts to a share of 0.17, so at the steady state g_z is calibrated to 1.2482 yielding a factor $\left(1 - \frac{1}{g_z}\right) \approx 0.17$. Regarding MP, we assume pure inflation targeting, i.e., $\phi_Y^z = 0$.[49] Remaining parameters are estimated.

Prior densities parameters are reported in Table 3. The choice of our prior densities draws on the literature, especially Smets & Wouters (2003) and Smets & Wouters (2007). Beginning with exogenous stochastic processes, we assume that innovations' SDs are distributed following inverted gamma processes and consider three homogeneous groups: (i) more loose distributions, i.e., with mean 0.1 and 2 degrees of freedom (df), for technology, money demand, preferences, investment and labor effort; (ii) markups shocks are distributed with mean 0.01 and 1 df; and (iii) government spending shocks that disturb very little $g_{z,t}$ are specified with mean 0.005 and 1 df. Moreover, all persistence parameters of AR(1) processes are beta-distributed with means 0.75 and SDs 0.1. Regarding policy, prior densities for the inflation targeting MP parameters where chosen as in the literature where the CB's reaction on inflation is normally distributed with mean 1.5 and SD 0.125. The prior on the parameter θ^z which measures a shift in the cost of adjusting the utilization rate of physical capital follows a normal distribution ($\mathcal{N}$) with mean 1 and SD of 0.15 (Adjemian *et al.* (2008) employ a gamma distribution). Moreover, the investment adjustment friction, Ψ^z, is normally distributed with mean 8 and SD equal to 2. Capital share is assumed to be distributed as beta (β) with mean 0.24 and standard error 0.05. Calvo wage stickiness is consistent with a prior probability α_W^z, beta-distributed with mean 0.5 and standard deviation 0.2 as it was suggested by Bils & Klenow (2004). Identical parameterization of the beta distribution is made for α_P^z. These priors mean that *a priori* hypotheses are that prices and wages are reset on average twice a year. The internal habit formation parameter is assumed to be beta-distributed with mean 0.4 and SD 0.2 (Adolfson *et al.* (2008)). Finally, the elasticity of substitution between home and foreign aggregates is inverted gamma distributed with mean 1.5 and 4 df (Adolfson *et al.* (2008)).

8.3 Characterization of posterior estimates

The set of posterior estimated parameters for our two subsample periods, namely, pre-EMU and EMU, is reported in Table 3. Posterior densities result from 250,000 replications (from were we disregard initial 50,000 values (20 percent) to avoid arbitrary results due to initialization).[50]

Beginning with the list of persistence parameters, we observe that: (i) technology process for Austria displays roughly the same value across subsamples (for example, $\rho_A^A = 0.61$ implies that the shock dies completely out in 2.58 quarters), whereas for the EA and the U.S. these are more persistent for the EMU period (for the EA the increase represents more than 12 percent); (ii) the MP persistence seems to be similar for Austria and the U.S., but slightly less persistent for the EA in the pre-EMU sample, whereas in the EMU regime estimates of the EA and the U.S. indicate less persistency (naturally we do not observe this parameter for Austria in the second period since its MP is managed by the ECB); (iii) estimates of government spending persistency are among the largest reported and remain robust across regimes with values ranging from 0.77 to 0.88 and, not surprisingly, estimates for Austria and the EA are quite similar; (iv) estimates of preference shock persistence are higher in the second regime —in the case of Austria it rises 13 percent, but for the EA and the U.S. variations represent more than 35 percent—; (v) estimates of investment persistency are found to be among the most persistent, in general these estimates marginally differ

[49] Schmitt-Grohe & Uribe (2005) and Paustian & von Hagen (2008) report that inflation targeting performs better in terms of welfare than the Taylor rule. In particular, Schmitt-Grohe & Uribe (2005) find that the simple rule that could sustain the Ramsey equilibrium, should also react to wage inflation with almost mute response to output gap. We do not try wage rules since CBs in practice are not explicitly targeting wage inflation.

[50] Mode vectors and checks as well as posterior distributions graphs are available on request from the authors.

across periods, with the exception of the Austrian estimate that goes down from 0.94 to 0.87; and (vi) estimates of the labor effort shock persistency seem to present similar patterns across regimes for Austria; however, the estimates increase in the EMU period for the EA and the U.S.: the latter's estimate is the one that varies the most (from 0.52 to 0.62).

We also report in Table 3 estimates of ARMA(1,1) markup processes. Autoregressive parts of these processes —estimates of persistency— range from 0.62 to 0.88 and are remarkably the same and robust for Austrian price and wage markups; however, we observe some changes across regimes in the other countries: (i) U.S. price markups estimates decrease from 0.88 to 0.75 and (ii) EA wage markups estimates decrease from 0.65 to 0.62. Regarding the parameters of the MA(1) representation, estimates do not display a clear pattern. Estimates for Austrian wages (but not prices) seem to be quite robust across regimes. We find a substantial increase in price MA parameters of markups for the U.S..

Regarding MP parameters, we find that OeNB and ECB are more sensitive to fluctuations of inflation than the FED. In fact, the period (effective) reaction parameter to inflation fluctuations results from $(1-\rho_R^z)\phi_\pi^z$, which in the pre-EMU sample amounts to values of 0.56 for Austria, 0.63 for the EA and 0.44 for the U.S.. In the EMU period, the latter two increase to 0.71 and 0.58, respectively. In accordance with Breuss & Rabitsch (2009), we find that ρ_R^U and ρ_R^E decrease in the EMU subsample. The shift in the utilization cost seems to be higher for Austria and the EA than for the U.S. which is observed close to 1. An interpretation valid for the U.S. is that as $r_z^k/\theta^z \rightarrow r_z^k$ (since $\theta^z \rightarrow 1$) it conveys evidence on a more perfect functioning of capital rental markets. Estimates of θ^z are 1.17 and 1.24 (about 20 percent higher than the prior) in the Euro Area and Austria, respectively. These values decrease in the EMU period to 1.08 and 1.14, respectively. Moreover, adjustment costs of new investment for all countries are estimated above the prior mean, especially in the pre-EMU sample (in the case of Austria, e.g., the estimate is 15.47 about two-times the prior mean). For the EMU subsample estimates are lower: 13.32 for Austria, 11.84 for the EA and 11.13 for the U.S.. Notice that the evidence points to lower frictions in investment in the EMU period for Austria and the EA (whereas for the U.S. remains the same) suggesting a more beneficial environment for investment in the EMU subsample. Moreover, estimates of nominal rigidities (Calvo parameters) of prices and wages lead us to calculate estimated durations of average price and wage contracts. This is accomplished for Austria employing the formula $\frac{1}{1-\alpha_P^A}$, which results in 2.41 quarters (2.94 in the EMU era), while for the EA and the U.S. these are 2.85 (3.28) and 4.96 (3.77) quarters, respectively. These results seem quite difficult to interpret, since one would expect that evidence on more rigid prices would be supported by data. In addition, the same method leads us to estimate (average) wage contracts durations of the three countries. For Austria, these are 5.18 and 7.96 quarters in the pre-EMU and EMU subsamples, for the EA these are 2.74 and 3.78 quarters and for the U.S. 4.27 and 5.3, respectively. Again, these results are quite surprising.[51] The general picture suggests larger nominal rigidities in wages in the EMU subsample which is to some extent worrisome for the EA due to the fact it necessitates wage flexibility to ease the adjustment of the real exchange rate. Moreover, internal habit formation estimates are quite stable and similar for all countries and these imply sizeable persistency in consumption. Finally, the elasticity of substitution η^z is (surprisingly) estimated lower than one indicating that home and imported variety bundles are complements. During the EMU period, these elasticities increase at least by a factor of 1.44 regardless the country considered (it is 1.63 in Austria, 1.49 in the EA and 1.44 for the U.S.).

Continuing with Table 3, SD estimates are also reported. These contribute to explain business cycle fluctuations which are driven by labor shocks, wage markup shocks, preference shocks and

[51] Probably the idea of similar prior distributions must be revised in future work to correct this pattern. Previous estimates of two country models suggest the opposite results: more rigidity is found in the EA than in the U.S.; nonetheless, we are not aware of evidence from three-country models.

investment shocks. In fact, labor effort shock (and not much wage markup shock), investment, technology and preference shocks seem to matter the most for the U.S. in the pre-EMU period (in the EMU period labor and technology shocks are less important, whereas the others remain marginally more relevant). Moreover, investment, labor effort and preference shocks are particularly important for the EA in the pre-EMU regime and still remain important during the EMU regime, though SD estimates have lower values. Finally, for Austria labor, MP and preference shocks seem to be most important.

Table 3: Bayesian estimation results

Parameters		Prior densities			Posteriors, Pre-EMU			Posteriors, EMU		
		Type	Mean	S.D.	Mean	0.05	0.95	Mean	0.05	0.95
Tech. shock persistence	ρ_A^A	β	0.75	0.1	0.591	0.481	0.706	0.606	0.483	0.730
Tech. shock persistence	ρ_A^E	β	0.75	0.1	0.569	0.423	0.723	0.640	0.525	0.757
Tech. shock persistence	ρ_A^U	β	0.75	0.1	0.570	0.408	0.736	0.615	0.469	0.763
MP persistence	ρ_R^A	β	0.75	0.1	0.668	0.609	0.726			
MP persistence	ρ_R^E	β	0.75	0.1	0.603	0.538	0.673	0.565	0.478	0.653
MP persistence	ρ_R^U	β	0.75	0.1	0.677	0.611	0.742	0.572	0.491	0.658
Gov. shock persistence	ρ_G^A	β	0.75	0.1	0.878	0.809	0.949	0.863	0.793	0.939
Gov. shock persistence	ρ_G^E	β	0.75	0.1	0.873	0.794	0.949	0.886	0.824	0.955
Gov. shock persistence	ρ_G^U	β	0.75	0.1	0.793	0.698	0.890	0.784	0.665	0.904
Pref. shock persistence	ρ_U^A	β	0.75	0.1	0.694	0.568	0.810	0.786	0.666	0.910
Pref. shock persistence	ρ_U^E	β	0.75	0.1	0.387	0.256	0.516	0.558	0.405	0.707
Pref. shock persistence	ρ_U^U	β	0.75	0.1	0.386	0.250	0.523	0.522	0.346	0.694
Inv. shock persistence	ρ_I^A	β	0.75	0.1	0.942	0.913	0.972	0.874	0.803	0.946
Inv. shock persistence	ρ_I^E	β	0.75	0.1	0.902	0.864	0.940	0.902	0.849	0.957
Inv. shock persistence	ρ_I^U	β	0.75	0.1	0.895	0.841	0.950	0.861	0.793	0.931
Labor shock persistence	ρ_L^A	β	0.75	0.1	0.769	0.693	0.851	0.763	0.638	0.889
Labor shock persistence	ρ_L^E	β	0.75	0.1	0.617	0.393	0.816	0.665	0.490	0.844
Labor shock persistence	ρ_L^U	β	0.75	0.1	0.517	0.327	0.722	0.619	0.475	0.769
MA price markup persistence	$\rho_{\mu_P}^A$	β	0.75	0.1	0.721	0.561	0.885	0.747	0.594	0.910
MA price markup persistence	$\rho_{\mu_P}^E$	β	0.75	0.1	0.726	0.570	0.893	0.759	0.602	0.923
MA price markup persistence	$\rho_{\mu_P}^U$	β	0.75	0.1	0.883	0.698	0.989	0.751	0.595	0.917
MA wage markup persistence	$\rho_{\mu_W}^A$	β	0.75	0.1	0.756	0.607	0.910	0.750	0.595	0.914
MA wage markup persistence	$\rho_{\mu_W}^E$	β	0.75	0.1	0.651	0.454	0.841	0.621	0.438	0.812
MA wage markup persistence	$\rho_{\mu_W}^U$	β	0.75	0.1	0.748	0.588	0.914	0.751	0.595	0.907
MA price markup parameter	ϑ_P^A	β	0.5	0.2	0.584	0.279	0.913	0.507	0.192	0.833
MA price markup parameter	ϑ_P^E	β	0.5	0.2	0.554	0.227	0.893	0.501	0.178	0.802
MA price markup parameter	ϑ_P^U	β	0.5	0.2	0.334	0.018	0.669	0.478	0.148	0.805
MA wage markup parameter	ϑ_W^A	β	0.5	0.2	0.491	0.155	0.814	0.491	0.156	0.818
MA wage markup parameter	ϑ_W^E	β	0.5	0.2	0.758	0.554	0.968	0.779	0.620	0.937
MA wage markup parameter	ϑ_W^U	β	0.5	0.2	0.513	0.184	0.837	0.495	0.162	0.815
MP reaction to inflation	ϕ_π^A	$\mathcal{N}$	1.5	0.125	1.683	1.377	1.971			
MP reaction to inflation	ϕ_π^E	$\mathcal{N}$	1.5	0.125	1.584	1.311	1.835	1.623	1.428	1.814
MP reaction to inflation	ϕ_π^U	$\mathcal{N}$	1.5	0.125	1.373	1.000	1.881	1.366	1.172	1.552
Shift param. utilization cost	θ^A	$\mathcal{N}$	1	0.15	1.178	0.952	1.395	1.083	0.849	1.322
Shift param. utilization cost	θ^E	$\mathcal{N}$	1	0.15	1.236	1.016	1.464	1.144	0.915	1.382
Shift param. utilization cost	θ^U	$\mathcal{N}$	1	0.15	1.005	0.761	1.247	0.988	0.745	1.235
Param. investment cost	Ψ^A	$\mathcal{N}$	8	2.5	15.471	12.441	18.804	13.320	10.191	16.449
Param. investment cost	Ψ^E	$\mathcal{N}$	8	2.5	13.435	10.074	16.385	11.847	8.609	15.147
Param. investment cost	Ψ^U	$\mathcal{N}$	8	2.5	11.633	8.126	15.047	11.138	7.911	14.419
Calvo price	α_P^A	β	0.5	0.15	0.586	0.513	0.653	0.660	0.603	0.717
Calvo price	α_P^E	β	0.5	0.15	0.649	0.591	0.714	0.695	0.635	0.754
Calvo price	α_P^U	β	0.5	0.15	0.799	0.625	0.954	0.735	0.658	0.814

Table 3: Bayesian estimation results (continuation)

Parameters (cont.)		Prior densities			Posteriors, Pre-EMU			Posteriors, EMU		
		Type	Mean	S.D.	Mean	0.05	0.95	Mean	0.05	0.95
Calvo wage	α_W^A	β	0.5	0.15	0.807	0.751	0.868	0.874	0.819	0.930
Calvo wage	α_W^E	β	0.5	0.15	0.636	0.530	0.741	0.735	0.632	0.844
Calvo wage	α_W^U	β	0.5	0.15	0.766	0.688	0.848	0.811	0.749	0.875
Habit formation	b^A	β	0.5	0.2	0.935	0.924	0.946	0.939	0.923	0.955
Habit formation	b^E	β	0.5	0.2	0.934	0.924	0.945	0.926	0.911	0.941
Habit formation	b^U	β	0.5	0.2	0.925	0.911	0.938	0.921	0.900	0.942
Inverse Frisch elast.	σ_L^A	$\mathcal{N}$	2	0.5	1.980	1.289	2.674	1.679	0.971	2.404
Inverse Frisch elast.	σ_L^E	$\mathcal{N}$	2	0.5	2.078	1.419	2.769	1.854	1.249	2.462
Inverse Frisch elast.	σ_L^U	$\mathcal{N}$	2	0.5	2.384	1.647	3.163	1.983	1.327	2.611
Elasticity of subst.	η^A	Γ^{-1}	1.5	4	0.367	0.270	0.461	0.596	0.356	0.831
Elasticity of subst.	η^E	Γ^{-1}	1.5	4	0.413	0.289	0.528	0.616	0.353	0.864
Elasticity of subst.	η^U	Γ^{-1}	1.5	4	0.365	0.274	0.455	0.524	0.338	0.700

Standard Errors of Shocks		Prior densities			Posteriors, Pre-EMU			Posteriors, EMU		
		Type	Mean	df	Mean	0.05	0.95	Mean	0.05	0.95
Technology	$\sigma_{\nu_Y^A}$	Γ^{-1}	0.1	2	0.019	0.015	0.024	0.025	0.019	0.032
Technology	$\sigma_{\nu_Y^E}$	Γ^{-1}	0.1	2	0.020	0.015	0.024	0.020	0.015	0.025
Technology	$\sigma_{\nu_Y^U}$	Γ^{-1}	0.1	2	0.044	0.022	0.068	0.036	0.021	0.051
Money demand	$\sigma_{\nu_R^A}$	Γ^{-1}	0.1	2	0.024	0.019	0.029			
Money demand	$\sigma_{\nu_R^E}$	Γ^{-1}	0.1	2	0.035	0.028	0.042	0.020	0.016	0.024
Money demand	$\sigma_{\nu_R^U}$	Γ^{-1}	0.1	2	0.023	0.018	0.028	0.021	0.016	0.026
Preferences	$\sigma_{\nu_U^A}$	Γ^{-1}	0.1	2	0.021	0.017	0.026	0.025	0.018	0.032
Preferences	$\sigma_{\nu_U^E}$	Γ^{-1}	0.1	2	0.036	0.029	0.043	0.025	0.019	0.031
Preferences	$\sigma_{\nu_U^U}$	Γ^{-1}	0.1	2	0.033	0.025	0.040	0.037	0.026	0.048
Investment	$\sigma_{\nu_I^A}$	Γ^{-1}	0.1	2	0.018	0.015	0.021	0.022	0.017	0.027
Investment	$\sigma_{\nu_I^E}$	Γ^{-1}	0.1	2	0.031	0.026	0.036	0.023	0.019	0.028
Investment	$\sigma_{\nu_I^U}$	Γ^{-1}	0.1	2	0.054	0.046	0.063	0.056	0.045	0.068
Labor effort	$\sigma_{\nu_L^A}$	Γ^{-1}	0.1	2	0.045	0.027	0.063	0.066	0.027	0.101
Labor effort	$\sigma_{\nu_L^E}$	Γ^{-1}	0.1	2	0.039	0.020	0.059	0.035	0.020	0.050
Labor effort	$\sigma_{\nu_L^U}$	Γ^{-1}	0.1	2	0.094	0.025	0.165	0.080	0.040	0.118
Price markup	$\sigma_{\nu_{\mu_P}^A}$	Γ^{-1}	0.01	1	0.005	0.003	0.007	0.005	0.002	0.007
Price markup	$\sigma_{\nu_{\mu_P}^E}$	Γ^{-1}	0.01	1	0.004	0.002	0.006	0.005	0.002	0.008
Price markup	$\sigma_{\nu_{\mu_P}^U}$	Γ^{-1}	0.01	1	0.032	0.003	0.065	0.008	0.002	0.014
Wage markup	$\sigma_{\nu_{\mu_W}^A}$	Γ^{-1}	0.01	1	0.007	0.002	0.012	0.009	0.002	0.016
Wage markup	$\sigma_{\nu_{\mu_W}^E}$	Γ^{-1}	0.01	1	0.089	0.003	0.132	0.093	0.046	0.153
Wage markup	$\sigma_{\nu_{\mu_W}^U}$	Γ^{-1}	0.01	1	0.009	0.002	0.018	0.008	0.002	0.015
Gov. spending	σ_{G^A}	Γ^{-1}	0.005	1	0.003	0.003	0.004	0.004	0.003	0.005
Gov. spending	σ_{G^E}	Γ^{-1}	0.005	1	0.003	0.003	0.004	0.004	0.003	0.004
Gov. spending	σ_{G^U}	Γ^{-1}	0.005	1	0.005	0.004	0.006	0.004	0.003	0.005

Note: Upper-indices A, E and U stand for Austria, the EA and the U.S., respectively. Moreover, 'df' is the abbreviation of degrees of freedom.

9 Simulation results

In this section we present simulations once the model is parameterized with estimated values. We examine in turn IRFs, the variance decomposition and welfare implications.

9.1 Impulse response functions

The DSGE model (71) is solved and simulated employing a second order approximation following Schmitt-Grohe & Uribe (2003)'s method in the neighborhood of the steady state of the model that is parameterized with estimated parameters. The policy function that solves the model depends also on variances of shocks:

$$\mathbf{y}_t = \mathbf{y} + \mathbf{A}\mathbf{y}_{t-1} + \mathbf{B}\mathbf{v}_t + \frac{1}{2}[\mathbf{y}'_{t-1}\mathbf{C}\mathbf{y}_{t-1} + \mathbf{v}'_t\mathbf{D}\mathbf{v}_t] + \mathbf{y}'_{t-1}\mathbf{E}\mathbf{v}_t + \mathbf{\Delta}\mathbf{\Sigma}_{\mathbf{v}}, \tag{78}$$

where coefficients on state (exogenous) variables are contained in $\mathbf{A}$ ($\mathbf{B}$), cross products of state (exogenous) variables are in $\mathbf{C}$ ($\mathbf{D}$) and cross-product of the state and exogenous variables are in $\mathbf{E}$. Finally, $\mathbf{\Delta}$ contains shocks' variances. Notice that this method allows us to depart from the certainty equivalence principle (that holds under first order approximation).[52]

This section briefly describes impulse response functions of various shocks of Austrian, EA and U.S. origin considered in our model. To facilitate the interpretation the size of the shock impulses are of one percent and we report responses for the two regimes considered, namely, pre-EMU and EMU regimes. The former is represented with responses without markers, while EMU subsample responses are denoted with them. Dashed black lines (- - - - -) represent Austria, dashed with dot blue lines(- . - . -) the EA and full magenta lines (——) the U.S..

Figures 1 to 3 illustrate IRFs calculated from shocks occurring in Austria, the EA and the U.S., respectively. Beginning with the first row in Figure 1, a one percent shock in technology makes Austrian output go up in both regimes as in Breuss & Rabitsch (2009). Due to spillover effects channeled through trade, the EA and the U.S. outputs also react positively, but these effects are much short-lived in comparison with Austria's GDP. In particular, notice that the estimated IRF for the EMU regime is shifted upward. Austrian consumption goes up in the EMU period, while it goes up firstly, but then turns negative in pre-EMU sample period. Foreign countries' consumption all go down (with the exception of the EA in the EMU regime). Investment goes up in all countries because of the loosening of MP managed by the CBs; clearly Austrian investment react the most. Finally, hours worked in Austria go down, which is consistent with the productivity shock that saves inputs, whereas in the foreign countries hours worked go in the opposite direction.

In the second row of Figure 1 we find the impulse responses of a MP shock that decreases the interest rates in the Euro Area (in this particular case we do not consider Austria, since its MP is unobservable during the second regime). As a result in the EMU subsample, Austrian and EA GDPs go similarly up, while in the pre-EMU period Austrian GDP does not change much due to the fact that the nominal exchange rate is flexible and it reacts as a shock absorber. The U.S. GDP goes down under both regimes, displaying a more deeply response in the pre-EMU period. Consumption patterns are quite persistent due to large habit formation parameter estimates, showing reactions that go in the same direction as GDP. Investment reacts positively

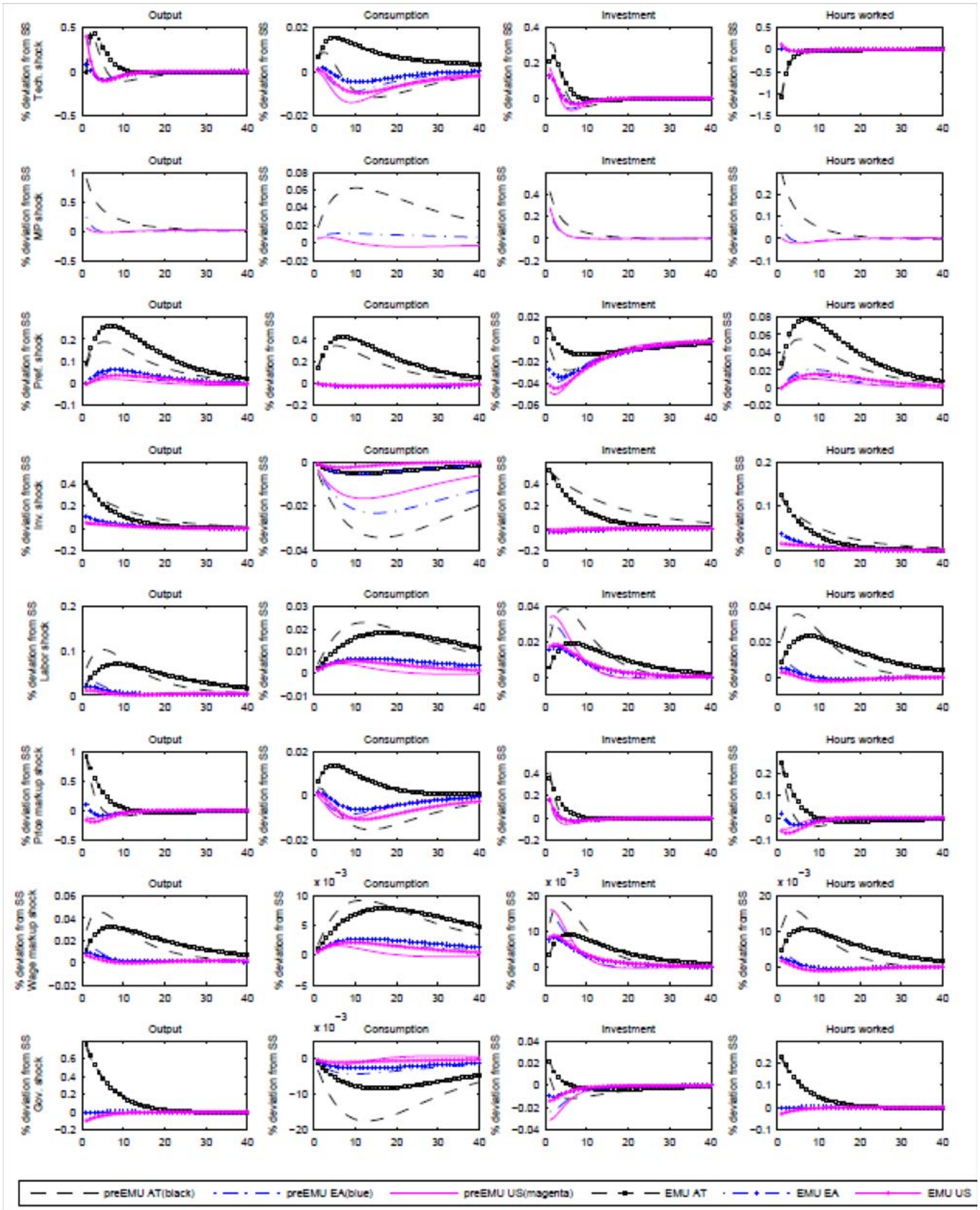
[52] Strictly speaking the procedure is not entirely correct, as an estimation of at least 2nd order approximation of the model must be taken to the the data. Such an estimation strategy involves the use of the Particle filter which would surely lead to different parameter results (and as a result to different numbers for welfare costs), see Fernandez-Villaverde & Rubio-Ramirez (2006).

Fernandez-Villaverde *et al.* (2006) have shown that 2nd order approximation errors in the solution of the model have first order effects on the likelihood function and error's magnitudes compound with the size of the sample. In conclusion the likelihood implied by the linearized model diverges from the likelihood implied by 2nd order and this, in turn, from the exact model.

Their proposed solution is simply not yet feasible for large scale models. This fact constrained us to employ our estimates obtained with a linear Kalman filter in parameterizing another model version, which is solved with 2nd a order approximation and ultimately it is simulated.

quite a bit to this interest rate drop, especially in the very short-run (until the 10th quarter) and then stabilizes. Hours worked display a very similar pattern as of GDP.

Figure 1: IRFs of shocks with origin in Austria



The third row of Figure 1 illustrates a one percent positive preference shock that occurs in

Austria. The impact on Austria is by large the most relevant in domestic GDP an also it spreads (not surprisingly) over the EA. The trade channel explains what we observe in the second chart, where solely consumption in Austria is fully responsive for this shock; the EA consumption does not react much; in fact, that response would not be easily distinguished from the U.S. consumption. Austrian investment reacts more positively in the EMU regime than in the pre-EMU. Again, the chart of hours worked depicts a picture very similar as of the one for GDP.

The fourth row of Figure 1 displays how is the response of GDP, consumption, investment and hours worked to a one percent shock in investment (the shock turns less expensive to install new investment).[53] Our results are very similar to those obtained by Breuss & Rabitsch (2009), though our specification slightly differs. The result is that Austrian investment goes up, while consumption goes down. These effects compensate, but since the investment one is more important, it dominates the expansion in the Austrian GDP. Again, hours worked depict a picture very similar as of the one for GDP.

The fifth row of Figure 1 illustrates a positive labor supply shock, in the sense that employees are less willing to enjoy their free time. This is one of the more interesting shocks to analyze since invariably consumption goes up. The autonomous MP pursued by the OeNB increases the interest rate so that investment in the pre-EMU period reacts more. If the same shock takes place, in the EMU regime our estimates show that the ECB is not as successful as before; this is reasonable since it sets MP for the whole EA. For that reason, adding investment and consumption effects, lead to an interesting result: Austrian output IRFs in the two samples cross around the 14th quarter. Hours worked increase in Austria, as it is expected.

The sixth and seventh rows of Figure 1 illustrate responses of price and wage markups shocks with origin in Austria. As usual, output goes up in Austria since these shocks can be rationalized as negative cost-push shocks. Austrian GDP responds positively in both scenarios, it is quite visible that it is a plus to count with an additional instrument, namely the interest rate (pre-EMU). MP is a useful device (materialized through an increasing the interest rate) to ease the expansion in GDP. Again, hours worked mirror the behavior of GDP.

Finally, the row at the bottom of Figure 1 illustrates responses of government spending shocks with origin in Austria, the government spending increases one percent (notice that we rescaled the shock accordingly, because in (37) the shock distorts the government-to-GDP ratio). We find that Austrian output goes up equally in both subsamples, leading to crowding out of private consumption, increasing investment and hours worked. EA's output remains very close to the steady state as well as consumption, investment and hours worked.

Next, in Figure 2 we illustrate IRFs with shocks originated in the EA. Briefly, we highlight the key differences with Figure 1. First, in the second row, notice that Austrian variables react very similarly as those of the EA, which is not surprisingly because the MP is set by the ECB, beyond the control of the OeNB. In addition, in the fourth row we observe that hours worked go up in Austria in response to incentives to save that materialize because the MP is common (notice the different reaction of pre-EMU and EMU variables).

Finally, Figure 3 illustrates IRFs with shocks originated in the U.S.. Again, we concentrate in differences with Figure 1. In the first row, the reaction in output for the EMU period is more pronounced that in the pre-EMU. In addition, in row 7, a shock in wage markup generates a hump-shaped reaction in output that is taller in the EMU period (in Figure 1 it is the reverse for Austria)

[53] Since the shock distorts investment decisions, the quadratic cost that quantifies installation costs plays a key role.

Figure 2: IRFs shocks with origin in the EA.

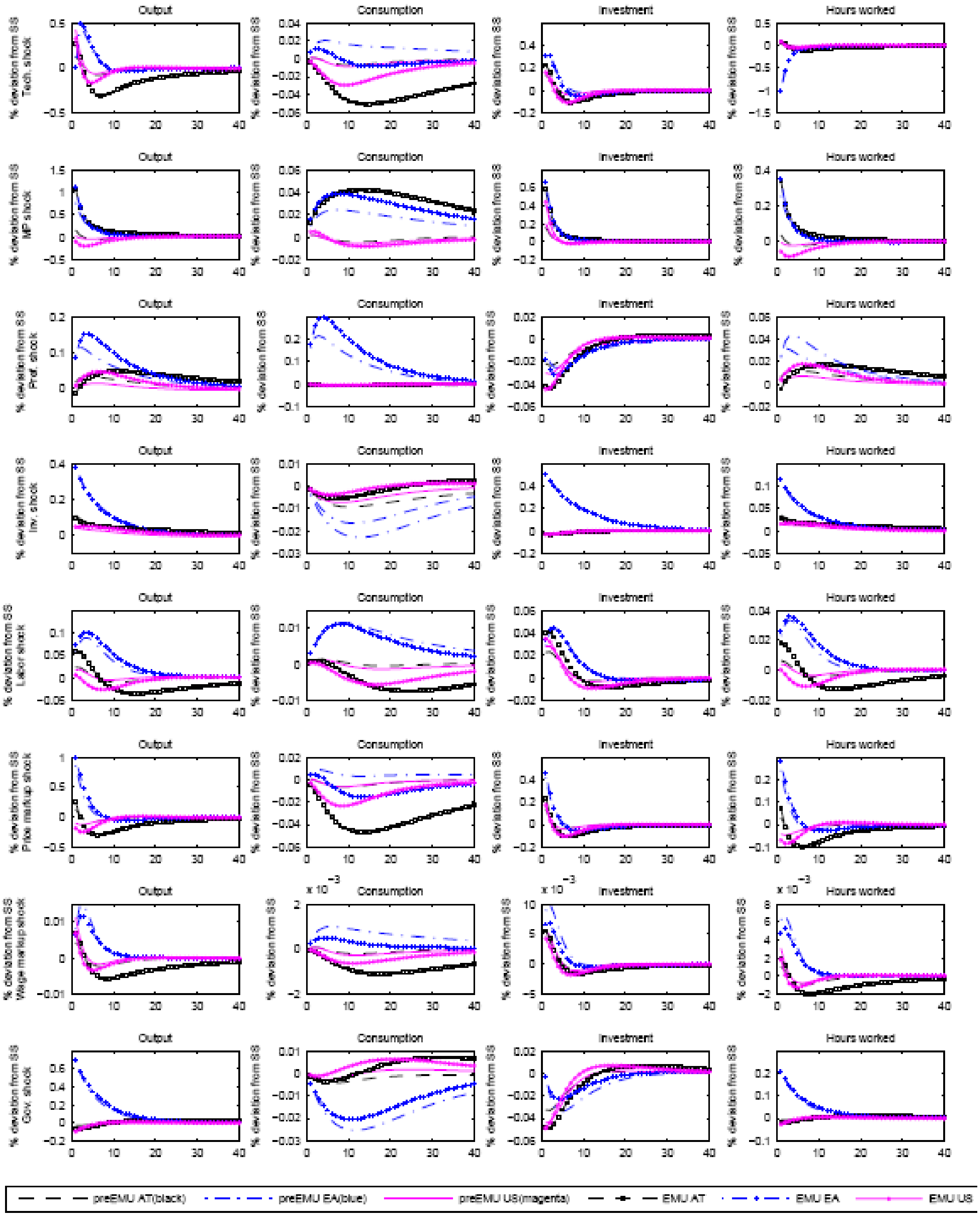
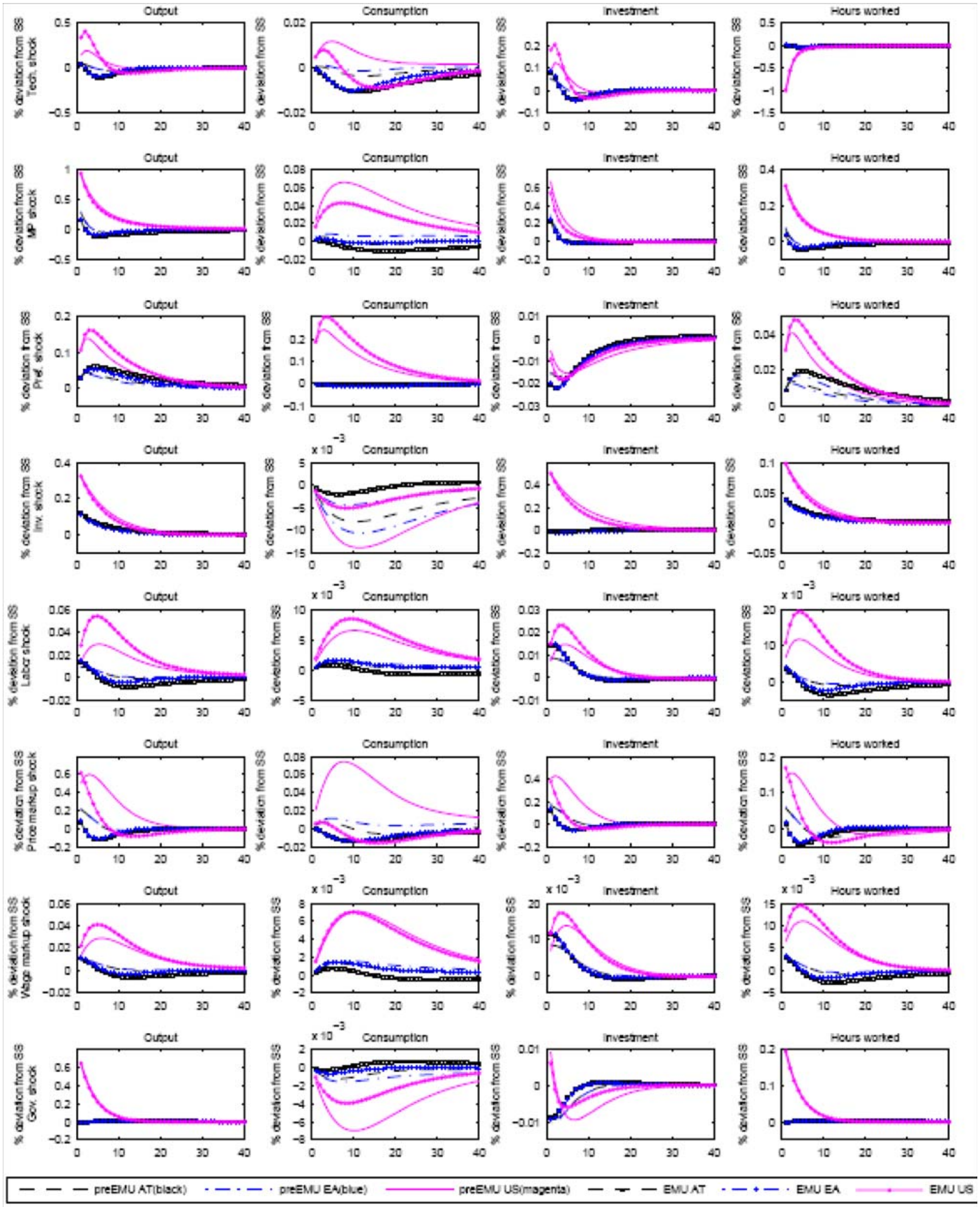


Figure 3: IRFs shocks with origin in the U.S.



9.2 Forecast error variance decomposition

The forecast error variance decomposition is reported in Tables 4 and 5 for horizons of 1 and 8 quarters.[54] Note that rows in these tables represent figures in percentage points and the (horizontal) sum adds up to one hundred. The general conclusion is that domestic shocks are quite important drivers for output, consumption and wage inflation not only in Austria but also in the EA and the U.S.. In contrast, foreign shocks are crucial for observed price inflation and observed nominal interest rates. Comparing consumption and investment variability, and their causes it is observed that Austrian business cycle is explained by foreign shocks to a greater extent than for the EA.

Beginning with Table 4, it is worth noticing particular countries' results. Firstly, consider the variability in Austrian GDP which to a large extent is driven by the MP shock regardless the subsample considered (note that in the EMU regime the MP shock is shown in 'foreign' since it originates in the EA). For output, also government shocks and technology shocks play key roles. Notice that technology shocks are much more relevant in the EMU regime (twice as much for the EA, by a factor of about 1.5 in Austria and the U.S.). Consumption fluctuations are to a large extent explained by preference shocks (more than 95 percent), displaying great stability across regimes. Investment variability in Austria, as in the other countries, is largely explained by investment shocks (the highest share is 45 percent for the EMU regime), followed by foreign (EMU) and domestic MP policy (pre-EMU). To analyze inflation variability we must differentiate both regimes. In effect, in the pre-EMU all the countries display as main sources MP shocks (pre-EMU), foreign and home technology shocks, which explain almost all the variability, whereas in the EMU period, technology shocks are very important for Austria (nearly 50 percent). Moreover, the labor effort shock substantially triggers wage inflation variability, reaching 65 percent in the EMU period. Other shocks worth mentioning are MP shocks (pre-EMU), technology shocks (in both regimes) and foreign shocks (note that this is relevant only for Austria with shocks occurring in the EA). Finally, the observed nominal interest rate variability (only available for the pre-EMU regime) is mostly caused by MP and productivity shocks, while nearly 35 percent is caused by foreign shocks.

The EA output has similar variability patterns as in Austria: the largest cause is the MP shock that accounts for 43 and 34 percent of the variability in the pre-EMU and EMU regimes, respectively. Moreover, to a lesser extent technology (specially in the EMU period), government and investment shocks are also important sources of GDP fluctuations. EA consumption variability is largely explained by preference shocks, a result that is in line with Breuss & Rabitsch (2009). Investment variability is triggered basically by investment, MP and technology shocks, whereas foreign shocks explain about 12 percent of the variability. Moreover, inflation variability draws heavily in MP and foreign shocks (pre-EMU) and technology shocks; however, there is a shift in the EMU period: we observe that the MP shock from explaining 61 percent drops to 22 percent. In addition, wage inflation variability is mostly (95 percent) caused by domestic shocks comprising labor effort, technology and MP shocks (this contrast with the sources of price inflation variability largely explained by foreign shocks). Finally, the observed interest rate variability is roughly explained by foreign shocks: 24 and 29 percent, in the pre-EMU and EMU periods, respectively. The rest is explained by domestic shocks including MP and technology shocks (the latter in the EMU period increases 50 percent).

Finally, consider the variability of U.S. observed variables. Regarding, output and consumption, we observe negligible dependency of foreign shocks, whereas other domestic shocks such as investment, productivity and MP are quite important in explaining GDP's variability (percentages remain quite stable in both regimes). Public expenditure is also important in the pre-EMU regime. Moreover, the U.S. consumption is again heavily influenced by preference shocks. Re-

[54] Other horizons were calculated, $h = 4$, 12 and 30, though for space reasons are not reported.

garding investment variability it is mainly caused by investment shocks (71 and 76 percent in the pre-EMU regime and in the EMU regime, respectively), technology and foreign shocks trigger to a large extent the remaining variability. Sources of U.S. inflation variability are very similar to the ones reported above for the EA: three shocks explain the bulk of the variability, MP, foreign and technology shocks. Wage inflation variance is largely explained by domestic shocks, we find that labor shock is very important, while technology and money demand shocks less important but sizeable. Finally, sources of variability in U.S. interest rates are foreign shocks (26-30 percent), technology, domestic MP shocks in that order of importance.

Table 5 presents the forecast variance decomposition for a forecast horizon of two years. Briefly, it points qualitatively to the same sources of fluctuations as just described. There are noticeable changes in some instances such as shares in investment shocks that explain investment variability (a pattern that repeats in all three countries). Other rows of Tables 4 and 5 are very similar, e.g. inflation and interest rate variability (all countries).

Table 4: Forecast error variance decomposition, t+1

Country	Shocks								
	Tech.	MP	Pref.	Inv.	Labor	P markup	W markup	Gov.	Foreign
AT									
$Y_{AT,t}^{preEMU}$	7.07	49.96	1.81	7.22	2.62	0.99	0.01	18.32	12.01
$Y_{AT,t}^{EMU}$	12.69		3.30	10.14	1.39	1.48	0.00	27.51	43.48
$C_{AT,t}^{preEMU}$	0.04	2.30	96.53	0.15	0.55	0.00	0.00	0.06	0.38
$C_{AT,t}^{EMU}$	0.16		98.82	0.01	0.28	0.01	0.00	0.01	0.71
$I_{AT,t}^{preEMU}$	11.49	29.60	0.11	39.09	1.15	0.50	0.00	0.01	18.05
$I_{AT,t}^{EMU}$	12.82		0.02	44.89	0.31	0.56	0.00	0.04	41.36
$\Pi_{AT,t}^{preEMU}$	16.74	46.63	0.06	0.03	0.23	1.17	0.00	0.02	35.11
$\Pi_{AT,t}^{EMU}$	47.93		0.26	0.06	0.82	2.38	0.00	0.09	48.46
$\Pi_{W,AT,t}^{preEMU}$	11.30	24.53	5.29	0.95	49.32	0.10	0.24	1.35	6.92
$\Pi_{W,AT,t}^{EMU}$	3.54		9.32	0.47	65.77	0.01	0.30	0.73	19.86
$R_{AT,t}^{preEMU}$	24.71	38.57	0.13	0.07	0.42	0.98	0.00	0.05	35.06
EA									
$Y_{EA,t}^{preEMU}$	10.31	43.47	1.73	15.46	2.35	0.82	0.02	15.88	9.95
$Y_{EA,t}^{EMU}$	21.18	34.17	0.91	10.11	2.30	2.16	0.05	18.88	10.23
$C_{EA,t}^{preEMU}$	0.32	0.46	98.05	0.23	0.13	0.00	0.00	0.19	0.63
$C_{EA,t}^{EMU}$	0.40	0.58	97.76	0.16	0.08	0.01	0.00	0.22	0.79
$I_{EA,t}^{preEMU}$	9.51	19.39	0.31	56.32	1.28	0.28	0.01	0.08	12.81
$I_{EA,t}^{EMU}$	16.90	24.55	0.49	43.49	1.27	0.77	0.02	0.16	12.36
$\Pi_{EA,t}^{preEMU}$	11.62	61.39	0.23	0.16	0.90	0.53	0.01	0.10	25.05
$\Pi_{EA,t}^{EMU}$	33.73	22.36	0.54	0.20	1.53	2.01	0.02	0.27	39.34
$\Pi_{W,EA,t}^{preEMU}$	14.35	29.02	6.36	1.56	43.54	0.03	0.22	1.52	3.41
$\Pi_{W,EA,t}^{EMU}$	22.46	11.47	7.68	1.05	49.87	0.07	0.49	1.91	5.01
$R_{EA,t}^{preEMU}$	21.63	45.74	0.53	0.36	2.09	0.59	0.01	0.20	28.85
$R_{EA,t}^{EMU}$	30.03	42.27	0.55	0.20	1.58	1.17	0.02	0.27	23.91
US									
$Y_{US,t}^{preEMU}$	21.35	22.78	1.49	24.93	1.86	0.59	0.00	19.71	7.27
$Y_{US,t}^{EMU}$	21.77	26.58	3.21	27.40	1.43	0.66	0.00	11.27	7.69
$C_{US,t}^{preEMU}$	0.28	1.46	97.09	0.34	0.34	0.00	0.00	0.04	0.45
$C_{US,t}^{EMU}$	0.13	0.51	99.01	0.04	0.15	0.00	0.00	0.00	0.17
$I_{US,t}^{preEMU}$	10.01	9.89	0.05	70.85	0.68	0.32	0.00	0.02	8.18
$I_{US,t}^{EMU}$	7.56	9.58	0.04	76.25	0.43	0.29	0.00	0.00	5.84
$\Pi_{US,t}^{preEMU}$	24.98	40.91	0.04	0.22	0.44	1.70	0.00	0.04	31.67
$\Pi_{US,t}^{EMU}$	26.88	32.61	0.20	0.41	0.62	1.08	0.00	0.02	38.17
$\Pi_{W,US,t}^{preEMU}$	16.14	14.72	1.47	2.79	60.97	0.08	0.12	1.08	2.63
$\Pi_{W,US,t}^{EMU}$	23.56	6.39	3.88	2.53	59.55	0.12	0.17	0.41	3.38
$R_{US,t}^{preEMU}$	39.74	27.61	0.08	0.50	0.94	1.36	0.00	0.09	29.68
$R_{US,t}^{EMU}$	35.55	35.29	0.30	0.59	0.92	1.01	0.00	0.04	26.31

Table 5: Forecast error variance decomposition, t+8

Country	Shocks								
	Tech.	MP	Pref.	Inv.	Labor	P markup	W markup	Gov.	Foreign
AT									
$Y_{AT,t}^{preEMU}$	5.63	43.03	4.84	8.64	5.56	0.69	0.01	18.11	13.48
$Y_{AT,t}^{EMU}$	9.35		9.70	9.14	5.02	1.02	0.01	23.45	42.31
$C_{AT,t}^{preEMU}$	0.05	3.97	93.22	0.42	1.40	0.00	0.00	0.14	0.79
$C_{AT,t}^{EMU}$	0.09		97.75	0.02	0.69	0.00	0.00	0.03	1.41
$I_{AT,t}^{preEMU}$	8.13	22.96	0.26	52.21	2.10	0.34	0.01	0.02	13.98
$I_{AT,t}^{EMU}$	10.22		0.16	51.71	1.31	0.45	0.00	0.03	36.12
$\Pi_{AT,t}^{preEMU}$	16.89	46.03	0.13	0.05	0.45	1.15	0.00	0.04	35.27
$\Pi_{AT,t}^{EMU}$	48.51		0.39	0.07	0.92	2.30	0.00	0.08	47.72
$\Pi_{W,AT,t}^{preEMU}$	11.95	26.74	5.18	1.30	42.06	0.18	0.20	1.45	10.92
$\Pi_{W,AT,t}^{EMU}$	3.14		8.91	0.54	52.99	0.03	0.23	0.66	33.49
$R_{AT,t}^{preEMU}$	22.72	39.60	0.52	0.20	1.82	0.89	0.01	0.14	34.11
EA									
$Y_{EA,t}^{preEMU}$	9.50	37.00	3.06	17.64	2.85	0.69	0.04	18.52	10.69
$Y_{EA,t}^{EMU}$	19.84	28.01	2.12	11.54	2.66	1.83	0.07	21.26	12.67
$C_{EA,t}^{preEMU}$	0.51	0.83	94.41	0.95	0.30	0.00	0.01	0.82	2.17
$C_{EA,t}^{EMU}$	0.37	1.04	94.23	0.56	0.13	0.00	0.01	0.74	2.91
$I_{EA,t}^{preEMU}$	7.21	14.42	0.42	65.46	1.19	0.21	0.01	0.21	10.87
$I_{EA,t}^{EMU}$	13.28	18.76	0.58	54.06	1.11	0.59	0.02	0.29	11.30
$\Pi_{EA,t}^{preEMU}$	11.54	60.81	0.31	0.28	0.94	0.53	0.01	0.20	25.40
$\Pi_{EA,t}^{EMU}$	33.23	21.96	0.70	0.34	1.58	1.98	0.02	0.47	39.73
$\Pi_{W,EA,t}^{preEMU}$	13.93	28.69	6.02	1.67	42.48	0.08	0.21	1.58	5.35
$\Pi_{W,EA,t}^{EMU}$	22.29	10.93	7.27	1.13	48.72	0.23	0.46	2.00	6.96
$R_{EA,t}^{preEMU}$	21.18	43.48	1.04	0.99	2.71	0.55	0.02	0.69	29.34
$R_{EA,t}^{EMU}$	30.04	40.34	1.02	0.51	1.93	1.12	0.03	0.72	24.29
US									
$Y_{US,t}^{preEMU}$	17.66	23.73	2.17	27.51	3.42	0.48	0.01	18.31	6.71
$Y_{US,t}^{EMU}$	18.71	25.02	5.45	28.80	3.37	0.53	0.01	9.92	8.18
$C_{US,t}^{preEMU}$	0.18	3.75	92.04	1.31	1.10	0.00	0.00	0.14	1.48
$C_{US,t}^{EMU}$	0.08	1.02	97.57	0.09	0.48	0.00	0.00	0.01	0.74
$I_{US,t}^{preEMU}$	7.38	7.86	0.09	76.81	0.86	0.23	0.00	0.04	6.73
$I_{US,t}^{EMU}$	6.03	7.55	0.08	80.13	0.70	0.22	0.00	0.00	5.28
$\Pi_{US,t}^{preEMU}$	25.10	40.45	0.06	0.31	0.55	1.69	0.00	0.05	31.80
$\Pi_{US,t}^{EMU}$	26.54	32.02	0.30	0.47	1.01	1.06	0.00	0.03	38.57
$\Pi_{W,US,t}^{preEMU}$	17.25	16.97	1.39	3.28	55.11	0.17	0.10	1.04	4.69
$\Pi_{W,US,t}^{EMU}$	26.86	6.90	4.01	3.08	52.67	0.29	0.15	0.41	5.62
$R_{US,t}^{preEMU}$	37.70	28.14	0.23	1.13	1.95	1.26	0.00	0.19	29.40
$R_{US,t}^{EMU}$	34.56	34.18	0.65	0.87	2.28	0.95	0.01	0.06	26.46

9.3 Welfare implications

We calculate the welfare cost of the allocation supported in the estimated model, according to Equation (64). The objective of this analysis provides us with a coherent indication of how far is the consumer from the efficient allocation, though these results must be interpreted with care as we stated in Footnote 52, on page 33. This utility gap, which is unobservable per se, is transformed in terms of steady-state consumption. Thus, to interpret figures that appear in Table 6 we remark that these are percentages of steady state consumption that nobody appropriates and is literally lost. Which factors reduce that utility gap? Our model has several nominal as well as real rigidities, which allow us to tackle the data. It is well known that if no rigidities were operative, our model would be hardly distinguishable from classical models, where the welfare cost is zero and MP is ineffective to boast aggregate demand (at any horizon). We analyze two scenarios: the first one is the benchmark model and the second one is a counterfactual, which assumes that nominal rigidities (Calvo price and wage estimated parameters) are cut by 50 percent.

Beginning with the estimated model, Table 6 reports welfare costs under our two regimes. For the pre-EMU period, the EA and Austria present welfare costs close to one percent of steady-state consumption (-0.91 percent for Austria and -1.01 percent for the EA), whereas the U.S. welfare costs is slightly higher: -1.52 percent of the steady-state consumption. As it is expect, in the second subsample welfare costs in the EA decrease. This can be interpreted as favorable evidence that indicate an improvement in the allocation during the EMU regime. The same result is obtained in the U.S. although there was no change in regime. Surprisingly, the welfare cost observed for Austria is higher; indicating that the typical consumer is worse off in terms of welfare. In particular, in the case of Austria the increase represents almost 20 percent, reaching a welfare cost of -1.08 percent. One explanation of these results might be that the EMU regime is characterized with data that spans too shortly compared with the pre-EMU period.

What would be the effect of a drop in nominal rigidities by 50 percent? This experiment is more valuable as it might appear because all three countries' welfare costs diminish. Why? Because a drop in the Calvo price and wage parameters is equivalent to more flexible pricing, therefore welfare cost decreases. In the EMU subsample, welfare costs are lower for all countries and Austria drops more than the EA, which is an interesting result. Likewise, in the EMU regime welfare costs go down more than in the EA. However, as the U.S. welfare cost reduces in the second regime as well, its use as a 'control' country is dubious. To make a fair comparison we normalize the welfare cost of Austria and EA in terms of the one observed for the U.S.. As a result, it becomes evident that the relative welfare cost of Austria and EA increase (despite the fact that in the EA, it decreased in absolute values).

Table 6: Conditional welfare costs in terms of steady state consumption

Country	All shocks				Relative to the U.S.			
	Estimated rigidities		Calvo param. $\downarrow$ 50%		Estimated rigidities		Calvo param. $\downarrow$ 50%	
	pre-EMU	EMU	pre-EMU	EMU	pre-EMU	EMU	pre-EMU	EMU
AT	-0.9103	-1.0855	-0.7110	-0.9191	0.598	1.109	0.527	1.07
EA	-1.0138	-0.8293	-0.9281	-0.8003	0.666	0.848	0.688	0.931
US	-1.5226	-0.9784	-1.3493	-0.8588				

10 Conclusions

In this paper we have studied a three-country DSGE model of the Austrian economy, an aggregate that accounts for the EA and the U.S. The model is estimated employing Bayesian methods on quarterly data covering the period from 1984:Q1 to 2007:Q4. That sample includes what is known as the 'Great Moderation' period widely studied for the U.S. Our main findings are summarized as follows. The sample is divided in two periods, namely, the pre-EMU and EMU regimes because of the evidence on different modes of the Austria and EA reported by Breuss & Rabitsch (2009).

The model is approximated up to the first order and the (linear) Kalman filter is employed to evaluate the likelihood. Further, Bayesian estimates reparameterize the model which is simulated employing second order approximation to the policy function. As a result we obtain IRFs, variance decompositions, FEVD and we calculate welfare costs.

Analyzing the FEVD, the general conclusion is that domestic shocks matter a lot for output and consumption in Austria as well as in the foreign countries; and in contrast, foreign shocks are important for observed price inflation and observed nominal interest rates. Furthermore, wage inflation variability is also explained fundamentally by domestic shocks. Comparing consumption and investment variability, for Austria and the EMU foreign shocks trigger more the former than the latter.

Our estimates of welfare costs under the two regimes are similar to the obtained in the literature. For the pre-EMU period, EA and Austria present welfare costs around one percent of steady-state consumption (-0.91 percent for the Austria and -1.01 percent for the EA), whereas the U.S. welfare cost is -1.52 percent. In the second subsample, the Austrian welfare cost is the only one that increases; in the case of the EA and the U.S. these drop to -0.83 and -0.98 percent, respectively. Also when compared with the U.S., it becomes evident that relative welfare cost of Austria and EA increase (despite in the EA, it decreased in absolute values).

We find interesting areas of further research such as the addition of exporters as a group to study the relevance of pass-through, the introduction of asset markets that are incomplete due to transaction costs derived from imperfect information when investment abroad and the evaluation of different policy rules in terms of welfare.

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A Appendix

A.1 Consumer's problem

Results presented in Section 3.1 implicitly draw on the Lagrangian for the consumer's problem. The relevant period t Lagrangian for an agent j^z that lives in country z ($z' \in Z^c$) can be written as:

$$\mathcal{L}_{z,t}^{period} = f\left(C_t^{j^z}, C_{t-1}^{j^z}, \frac{M_t^{j^z}}{P_t}, \frac{M_{t-1}^{j^z}}{P_{t-1}}, W_t^{j^z}, B_z^{j^z}\left(s^{t+h+1}\right), B_z^{j^z}\left(s^{t+h}\right), B_{z'}^{j^z}\left(s^{t+h+1}\right), B_{z'}^{j^z}\left(s^{t+h}\right), K_{z,t}, \Lambda_{z,t}^1, \Lambda_{z,t}^2\right),$$

which takes the form $\mathcal{L}_{z,t} = E_t\left[\sum_{h=0}^{\infty}\beta^{t+h}\mathcal{L}_{z,t+h}^{period}\right]$ if we consider her complete life period. Formally, $\mathcal{L}_{z,t}$ equals

$$E_t\left[\begin{array}{c} \sum_{h=0}^{\infty}\beta^{t+h}U\left(C_{t+h}^{j^z}, C_{t+h-1}^{j^z}, \frac{M_{t+h}^{j^z}}{P_{z,t+h}}, W_{t+h}^{j^z}\right) + \sum_{h=0}^{\infty}\beta^{t+h}\Lambda_{z,t+h}^1 \\ \times \\ \left(\begin{array}{c} B_z^{j^z}\left(s^{t+h}\right) + \sum_{z'\in Z^c} S_{z,z',t}B_{z'}^{j^z}\left(s^{t+h}\right) + M_{t+h-1}^{j^z} + \left(1-\tau_w^z\right)\mathcal{S}_W^z W_{t+h}^{j^z} L_{t+h}^{j^z} + T_{t+h}^{j^z} - TX_{t+h}^{j^z} \\ + \left(1-\tau_k^z\right)\left(\left(R_{t+h}^k\right)^{j^z} u_{t+h}^{j^z} K_{t+h-1}^{j^z} - P_{z,t+h}\Phi\left(u_{t+h}^{j^z}\right)K_{t+h-1}^{j^z}\right) - Q_z\left(s^{t+h+1}, s^{t+h}\right)B_z^{j^z}\left(s^{t+h+1}\right) \\ + \mathsf{Pr}_{t+h}^{j^z} - \left(\sum_{z'\in Z^c} S_{z,z',t}Q_{z'}\left(s^{t+h+1}, s^t\right)B_{z'}^{j^z}\left(s^{t+h+1}\right)\right) - M_{t+h}^{j^z} - P_{z,t+h}\left(C_{t+h}^{j^z} + I_{t+h}^{j^z}\right) \end{array}\right) \\ + \\ \sum_{h=0}^{\infty}\beta^{t+h}\Lambda_{z,t+h}^2\left(\left(1-\delta^z\right)K_{t+h-1}^{j^z} + \varepsilon_{I,t+h}^z I_{t+h}^{j^z} - \frac{\Psi}{2}\left(\frac{\varepsilon_{I,t+h}^z I_{t+h}^{j^z}}{K_{t+h-1}^{j^z}} - \delta^z\right)^2 K_{t+h-1}^{j^z} - K_{t+h}^{j^z}\right) \end{array}\right], \tag{79}$$

where choice variables are: $C_{t+h}^{j^z}$, $M_{t+h}^{j^z}$, $L_{t+h}^{j^z}$ ($W_{t+h}^{j^z}$ only if consumers are wage-setters in the labor market), $B_z^{j^z}\left(s^{t+h+1}\right)$, $B_{z'}^{j^z}\left(s^{t+h+1}\right)$, $B_{z''}^{j^z}\left(s^{t+h+1}\right)$, $K_{z,t+h}^{j^z}$, $I_t^{j^z}$.

Considering the utility function given in Equation (4) we derive the following partial derivatives with respect to consumption, money balances, labor supply (in the case of competitive labor markets) and the optimal wage:

$$\begin{aligned} U_{C_t^{j^z}} &= \left(C_t^{j^z} - h^z C_{t-1}^{j^z}\right)^{-\sigma_C^z}, \\ U_{C_{t-1}^{j^z}} &= \left(C_t^{j^z} - h^z C_{t-1}^{j^z}\right)^{-\sigma_C^z}(-h^z), \\ U_{M_t^{j^z}} &= \varkappa_M^z \sigma_M^z \left(\frac{M_t^{j^z}}{P_{z,t}}\right)^{\sigma_M^z - 1}\frac{1}{P_{z,t}}, \end{aligned}$$

under competitive labor markets, the partial derivative is $U_{L_t^{j^z}} = -\varepsilon_{L,t}^z\left(L_t^{j^z}\right)^{\sigma_L^z}$, while for noncompetitive markets $U_{W_t^{j^z}} = -\varepsilon_{L,t}^z\left(L_t^{j^z}\right)^{\sigma_L^z}\left[\sigma_W^z\left(\frac{W_t^{j^z}}{W_{z,t}}\right)^{-\sigma_W^z} L_{z,t}\left(\frac{W_t^{j^z}}{W_{z,t}}\right)^{-1}\left(\frac{1}{W_{z,t}}\right)\right]$, or

$$U_{W_t^{j^z}} = -\varepsilon_{L,t}^z \sigma_W^z \left(L_t^{j^z}\right)^{1+\sigma_L^z}\left(W_t^{j^z}\right)^{-1}.$$

The FOCs w.r.t. consumption, capital, investment and utilization rate are as follows:

$$\Lambda^1_{z,t} = \frac{\left(C^{j^z}_t - h^z C^{j^z}_{t-1}\right)^{-\sigma^z_C} + \beta E_t\left[(-h^z)\left(C^{j^z}_{t+1} - h^z C^{j^z}_t\right)^{-\sigma^z_C}\right]}{P_{z,t}},$$

$$\Lambda^2_{z,t} = \beta E_t \left\{ \begin{array}{c} \Lambda^1_{z,t+1}\left(1-\tau^z_k\right)\left(\left(R^k_{t+1}\right)^{j^z} u^{j^z}_{t+1} - P_{z,t+1}\Phi\left(u^{j^z}_{t+1}\right)\right) \\ +\Lambda^2_{z,t+1}\left[(1-\delta^z) - \frac{\Psi}{2}\left(\frac{\varepsilon^z_{I,t+1} I^{j^z}_{t+1}}{K^{j^z}_t} - \delta^z\right)^2 + \Psi\left(\frac{\varepsilon^z_{I,t+1} I^{j^z}_{t+1}}{K^{j^z}_t} - \delta^z\right)\frac{\varepsilon^z_{I,t+1} I^{j^z}_{t+1}}{K^{j^z}_t}\right] \end{array} \right\},$$

$$\Lambda^1_{z,t} P_{z,t} = \Lambda^2_{z,t}\varepsilon^z_{I,t} - \Lambda^2_{z,t}\Psi\left(\frac{\varepsilon^z_{I,t} I^{j^z}_t}{K^{j^z}_{t-1}} - \delta^z\right)\frac{\varepsilon^z_{I,t}}{K^{j^z}_{t-1}},$$

$$\left(R^k_t\right)^{j^z} = P_{z,t}\Phi'\left(u^{j^z}_t\right).$$

Replace (2) into the (modified) Lagrangian (79), specialized taking into account the utility function (4):

$$\mathrm{E}_t \left[\begin{array}{c} \sum_{h=0}^{\infty} \left(\alpha^z_W \beta\right)^h \left[... - \frac{\varepsilon^z_{L,t+h}}{1+\sigma^z_L} \left[\left(\frac{\tilde{W}^{j^z}_t}{W_{z,t+h}}\right)^{-\sigma^z_W} L_{z,t+h} \right]^{1+\sigma^z_L} \right] \\ +\sum_{h=0}^{\infty} \left(\alpha^z_W \beta\right)^h \Lambda^1_{z,t+h} \left(... + \left(1-\tau^z_w\right) \mathcal{S}^z_W \tilde{W}^{j^z}_t \left(\frac{\tilde{W}^{j^z}_t}{W_{z,t+h}}\right)^{-\sigma^z_W} L_{z,t+h} + ... \right) \end{array} \right]$$

Differentiation w.r.t. $\tilde{W}^{j^z}_t$ yields the following optimality condition:

$$\mathrm{E}_t \left[\sum_{h=0}^{\infty} \left(\alpha^z_W \beta\right)^h \left(-\varepsilon^z_{L,t+h}\right) \left[\left(\frac{\tilde{W}^{j^z}_t}{W_{z,t+h}}\right)^{-\sigma^z_W} L_{z,t+h} \right]^{\sigma^z_L} \left(-\sigma^z_W\right) \left(\frac{\tilde{W}^{j^z}_t}{W_{z,t+h}}\right)^{-\sigma^z_W} \left(\frac{\tilde{W}^{j^z}_t}{W_{z,t+h}}\right)^{-1} \frac{L_{z,t+h}}{W_{z,t+h}} \right] +$$
$$\mathrm{E}_t \left[\sum_{h=0}^{\infty} \left(\alpha^z_W \beta\right)^h \Lambda^1_{z,t+h} \left(1-\tau^z_w\right) \mathcal{S}^z_W \left(1-\sigma^z_W\right) \left(\frac{\tilde{W}^{j^z}_t}{W_{z,t+h}}\right)^{-\sigma^z_W} L_{z,t+h} \right] = 0,$$

multiplying by $\left(\tilde{W}^{j^z}_t\right)^{\sigma^z_W}$ and operating algebraically we obtain:

$$\mathrm{E}_t \left[\sum_{h=0}^{\infty} \left(\alpha^z_W \beta\right)^h \varepsilon^z_{L,t+h} \left[\left(\frac{\tilde{W}^{j^z}_t}{W_{z,t+h}}\right)^{-\sigma^z_W} L_{z,t+h} \right]^{\sigma^z_L} \sigma^z_W \left(W_{z,t+h}\right)^{\sigma^z_W} \left(\tilde{W}^{j^z}_t\right)^{-1} L_{z,t+h} \right] +$$
$$\mathrm{E}_t \left[\sum_{h=0}^{\infty} \left(\alpha^z_W \beta\right)^h \Lambda^1_{z,t+h} \left(1-\tau^z_w\right) \mathcal{S}^z_W \left(1-\sigma^z_W\right) \left(W_{z,t+h}\right)^{\sigma^z_W} L_{z,t+h} \right] = 0.$$

Operating in the previous expression and isolating $\left(\tilde{W}^{j^z}_t\right)^{1+\sigma^z_W \sigma^z_L}$:

$$\sigma^z_W \left(\tilde{W}^{j^z}_t\right)^{-1-\sigma^z_W \sigma^z_L} \mathrm{E}_t \left[\sum_{h=0}^{\infty} \left(\alpha^z_W \beta\right)^h \varepsilon^z_{L,t+h} \left(W_{z,t+h}\right)^{\sigma^z_W\left(1+\sigma^z_L\right)} \left(L_{z,t+h}\right)^{1+\sigma^z_L} \right] =$$
$$\frac{\left(1-\tau^z_w\right)\mathcal{S}^z_W}{\sigma^z_W - 1} \mathrm{E}_t \left[\sum_{h=0}^{\infty} \left(\alpha^z_W \beta\right)^h \Lambda^1_{z,t+h} \left(W_{z,t+h}\right)^{\sigma^z_W} L_{z,t+h} \right],$$

$$\left(\tilde{W}_t^{j^z}\right)^{1+\sigma_W^z\sigma_L^z} = \frac{\mu_W^z}{\left(1-\tau_w^z\right)\mathcal{S}_W^z}\frac{\mathrm{E}_t\left[\sum_{h=0}^{\infty}\left(\alpha_W^z\beta\right)^h\varepsilon_{L,t+h}^z\left(W_{z,t+h}\right)^{\sigma_W^z\left(1+\sigma_L^z\right)}\left(L_{z,t+h}\right)^{1+\sigma_L^z}\right]}{\mathrm{E}_t\left[\sum_{h=0}^{\infty}\left(\alpha_W^z\beta\right)^h\Lambda_{z,t+h}^1\left(W_{z,t+h}\right)^{\sigma_W^z}L_{z,t+h}\right]}. \tag{80}$$

To derive **real optimal wages** $\widetilde{rw}_t^{j^z} \equiv \tilde{W}_t^{j^z}/P_{z,t}$, so that the numerator of Equation (80) remains the same, while $P_{z,t}^{1+\sigma_W^z\sigma_L^z}$ multiplies its denominator. Since $\Lambda_{z,t+h}^1 = \tilde{\Lambda}_{z,t+h}^1/\left[P_{z,t+h}\right]$ and $W_{z,t+h}/P_{z,t+h} = w_{z,t+h}$, it follows:

$$\left(\widetilde{rw}_t^{j^z}\right)^{1+\sigma_W^z\sigma_L^z} = \frac{\sigma_W^z}{\left(1-\tau_w^z\right)\left(\sigma_W^z-1\right)}\frac{1}{P_{z,t}^{1+\sigma_W^z\sigma_L^z}}\frac{\mathrm{E}_t\left[\sum_{h=0}^{\infty}\left(\alpha_W^z\beta\right)^h\varepsilon_{L,t+h}^z\left(W_{z,t+h}\right)^{\sigma_W^z\left(1+\sigma_L^z\right)}\left(L_{z,t+h}\right)^{1+\sigma_L^z}\right]}{\mathrm{E}_t\left[\sum_{h=0}^{\infty}\left(\alpha_W^z\beta\right)^h\frac{\tilde{\Lambda}_{z,t+h}^1}{P_{z,t+h}}\left(W_{z,t+h}\right)^{\sigma_W^z}L_{z,t+h}\right]}$$

$$= \frac{\sigma_W^z}{\left(1-\tau_w^z\right)\left(\sigma_W^z-1\right)}\frac{1}{P_{z,t}^{\sigma_W^z\sigma_L^z}}\frac{\mathrm{E}_t\left[\sum_{h=0}^{\infty}\left(\alpha_W^z\beta\right)^h\varepsilon_{L,t+h}^z\left(W_{z,t+h}\right)^{\sigma_W^z\left(1+\sigma_L^z\right)}\left(L_{z,t+h}\right)^{1+\sigma_L^z}\right]}{\mathrm{E}_t\left[\sum_{h=0}^{\infty}\left(\alpha_W^z\beta\right)^h\frac{\tilde{\Lambda}_{z,t+h}^1}{\Pi_{z,t+h}}\left(W_{z,t+h}\right)^{\sigma_W^z}L_{z,t+h}\right]}$$

On the RHS, inside the numerator's summation multiply and divide by $\mathrm{E}_t\left[P_{z,t+h}^{\sigma_W^z\left(1+\sigma_L^z\right)}\right]$, and similarly for the denominator, but by $\mathrm{E}_t\left[P_{z,t+h}^{\sigma_W^z}\right]$ instead, so that we get:

$$\left(\widetilde{rw}_t^{j^z}\right)^{1+\sigma_W^z\sigma_L^z} = \frac{\sigma_W^z}{\left(1-\tau_w^z\right)\left(\sigma_W^z-1\right)}\frac{1}{P_{z,t}^{\sigma_W^z\sigma_L^z}}\frac{\mathrm{E}_t\left[\sum_{h=0}^{\infty}\left(\alpha_W^z\beta\right)^h\varepsilon_{L,t+h}^z\frac{P_{z,t+h}^{\sigma_W^z\left(1+\sigma_L^z\right)}}{P_{z,t+h}^{\sigma_W^z\left(1+\sigma_L^z\right)}}\left(W_{z,t+h}\right)^{\sigma_W^z\left(1+\sigma_L^z\right)}\left(L_{z,t+h}\right)^{1+\sigma_L^z}\right]}{\mathrm{E}_t\left[\sum_{h=0}^{\infty}\left(\alpha_W^z\beta\right)^h\frac{\tilde{\Lambda}_{z,t+h}^1}{\Pi_{z,t+h}}\frac{P_{z,t+h}^{\sigma_W^z}}{P_{z,t+h}^{\sigma_W^z}}\left(W_{z,t+h}\right)^{\sigma_W^z}L_{z,t+h}\right]}$$

operating,

$$\left(\widetilde{rw}_t^{j^z}\right)^{1+\sigma_W^z\sigma_L^z} = \frac{\sigma_W^z}{\left(1-\tau_w^z\right)\left(\sigma_W^z-1\right)}\frac{\mathrm{E}_t\left[\sum_{h=0}^{\infty}\left(\alpha_W^z\beta\right)^h\varepsilon_{L,t+h}^zP_{z,t+h}^{\sigma_W^z}\Pi_{z,t+h}^{\sigma_W^z\sigma_L^z}\left(w_{z,t+h}\right)^{\sigma_W^z\left(1+\sigma_L^z\right)}\left(L_{z,t+h}\right)^{1+\sigma_L^z}\right]}{\mathrm{E}_t\left[\sum_{h=0}^{\infty}\left(\alpha_W^z\beta\right)^h\frac{\tilde{\Lambda}_{z,t+h}^1}{\Pi_{z,t+h}}P_{z,t+h}^{\sigma_W^z}\left(w_{z,t+h}\right)^{\sigma_W^z}L_{z,t+h}\right]}$$

finally, multiplying and dividing the RHS by $\left[P_{z,t}\right]^{\sigma_W^z}$ and introducing them into the summation to get rid of prices, we obtain:

$$\left(\widetilde{rw}_t^{j^z}\right)^{1+\sigma_W^z\sigma_L^z} = \frac{\sigma_W^z}{\left(1-\tau_w^z\right)\left(\sigma_W^z-1\right)}\frac{\mathrm{E}_t\left[\sum_{h=0}^{\infty}\left(\alpha_W^z\beta\right)^h\varepsilon_{L,t+h}^z\Pi_{z,t+h}^{\sigma_W^z\left(1+\sigma_L^z\right)}\left(w_{z,t+h}\right)^{\sigma_W^z\left(1+\sigma_L^z\right)}\left(L_{z,t+h}\right)^{1+\sigma_L^z}\right]}{\mathrm{E}_t\left[\sum_{h=0}^{\infty}\left(\alpha_W^z\beta\right)^h\tilde{\Lambda}_{z,t+h}^1\Pi_{z,t+h}^{\sigma_W^z-1}w_{z,t+h}^{\sigma_W^z}L_{z,t+h}\right]}. \tag{81}$$

A.2 Intratemporal allocation

The consumer derives utility from a bundle of consumption goods. Given this reference bundle, the representative consumer optimally allocates goods' varieties such that the expenditure is minimized. Throughout all this subsection we **drop** the t-subindex because there are no dynamics involved in the subsequent problems. It should be noticed that the tax effect is irrelevant because $(1+\tau^z)$ applies to all varieties and aggregates —consumption and investment—, the same applies to all intratemporal problems in this section).

A.2.1 Private consumption

The total consumption expenditure in country z is $P_z C^{j^z} = P_{z,z}C_z^{j^z} + \sum_{z'\in Z^c} P_{z,z'}C_{z'}^{j^z}$, for all $j^z \in [0,1]$.[55] In addition, we assume that the country z consumption shares are distributed in the unit interval as follows:

$$\underbrace{+\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!+}_{n_{z1}}\underbrace{\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!+}_{n_{z2}}\underbrace{\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!+}_{1-(n_{z1}+n_{z2})}$$

The consumer's problem turns out to be:

$$\min P_{z,z}C_z^{j^z} + \textstyle\sum_{z'\in Z^c} P_{z,z'}C_{z'}^{j^z}, \text{ st. (19).}$$

Notice that the bundle is the one that was defined in Equation (19) when $\eta^z \to 1$. Thus, the Lagrangian is:

$$\begin{aligned}\mathcal{L}_z &= P_{z,z}C_z^{j^z} + \textstyle\sum_{z'\in Z^c} P_{z,z'}C_{z'}^{j^z} \\ &+ \lambda_z \left\{ C^{j^z} - \left[(n_{z1})^{\frac{1}{\eta^z}} \left(C_z^{j^z}\right)^{\frac{\eta^z-1}{\eta^z}} + (n_{z2})^{\frac{1}{\eta^z}} \left(C_{z'}^{j^z}\right)^{\frac{\eta^z-1}{\eta^z}} + (1-n_{z1}-n_{z2})^{\frac{1}{\eta^z}} \left(C_{z''}^{j^z}\right)^{\frac{\eta^z-1}{\eta^z}} \right]^{\frac{\eta^z}{\eta^z-1}} \right\}\end{aligned}$$

FOCs:

$$\begin{aligned}
\mathcal{L}_{C_z} &= 0 \Leftrightarrow P_{z,z} - \lambda_z \left(\frac{\eta^z}{\eta^z-1}\right) []^{\frac{1}{\eta^z-1}} (n_{z1})^{\frac{1}{\eta^z}} \left(\frac{\eta^z-1}{\eta^z}\right) \left(C_z^{j^z}\right)^{\frac{-1}{\eta^z}} = 0 \\
\mathcal{L}_{C_{z'}} &= 0 \Leftrightarrow P_{z,z'} - \lambda_z \left(\frac{\eta^z}{\eta^z-1}\right) []^{\frac{1}{\eta^z-1}} (n_{z2})^{\frac{1}{\eta^z}} \left(\frac{\eta^z-1}{\eta^z}\right) \left(C_{z'}^{j^z}\right)^{\frac{-1}{\eta^z}} = 0 \\
\mathcal{L}_{C_{z''}} &= 0 \Leftrightarrow P_{z,z''} - \lambda_z \left(\frac{\eta^z}{\eta^z-1}\right) []^{\frac{1}{\eta^z-1}} (1-n_{z1}-n_{z2})^{\frac{1}{\eta^z}} \left(\frac{\eta^z-1}{\eta^z}\right) \left(C_{z''}^{j^z}\right)^{\frac{-1}{\eta^z}} = 0 \\
\mathcal{L}_{\lambda_z} &= 0 \Leftrightarrow C^{j^z} = \left[(n_{z1})^{\frac{1}{\eta^z}} \left(C_z^{j^z}\right)^{\frac{\eta^z-1}{\eta^z}} + (n_{z2})^{\frac{1}{\eta^z}} \left(C_{z'}^{j^z}\right)^{\frac{\eta^z-1}{\eta^z}} + (1-n_{z1}-n_{z2})^{\frac{1}{\eta^z}} \left(C_{z''}^{j^z}\right)^{\frac{\eta^z-1}{\eta^z}} \right]^{\frac{\eta^z}{\eta^z-1}}
\end{aligned} \tag{82}$$

calculating the ratio $\mathcal{L}_{C_z}/\mathcal{L}_{C_{z'}}$ and $\mathcal{L}_{C_{z''}}/\mathcal{L}_{C_{z'}}$ to get rid of λ_z yields:

$$\frac{P_{z,z}}{P_{z,z'}} = \frac{(n_{z1})^{\frac{1}{\eta^z}} \left(C_z^{j^z}\right)^{\frac{-1}{\eta^z}}}{(n_{z2})^{\frac{1}{\eta^z}} \left(C_{z'}^{j^z}\right)^{\frac{-1}{\eta^z}}} \quad \text{and} \quad \frac{P_{z,z''}}{P_{z,z'}} = \frac{(1-n_{z1}-n_{z2})^{\frac{1}{\eta^z}}}{(n_{z2})^{\frac{1}{\eta^z}}} \left(\frac{C_{z'}^{j^z}}{C_{z''}^{j^z}}\right)^{\frac{-1}{\eta^z}}$$

[55] If the law of one price would apply, $P_{z,z'} = S_{z,z'}P_{z',z'}$. The price $P_{z,z'}$ is interpreted as the price quoted in country z or an aggregate of goods produced in country z'.

replacing $C^{j^z}_{z'} = \frac{n_{z2}}{n_{z1}}\left(\frac{P_{z,z'}}{P_{z,z}}\right)^{-\eta^z} C^{j^z}_{z}$ and $C^{j^z}_{z''} = \frac{1-n_{z1}-n_{z2}}{n_{z2}}\left(\frac{P_{z,z''}}{P_{z,z'}}\right)^{-\eta^z} C^{j^z}_{z'}$ into the budget constraint (82), we get:

$$
\begin{aligned}
\left(C^{j^z}\right)^{\frac{\eta^z-1}{\eta^z}} &= \left[\begin{array}{c} (n_{z1})^{\frac{1}{\eta^z}}\left(C^{j^z}_z\right)^{\frac{\eta^z-1}{\eta^z}} + (n_{z2})^{\frac{1}{\eta^z}}\left(\frac{n_{z2}}{n_{z1}}\left(\frac{P_{z,z'}}{P_{z,z}}\right)^{-\eta^z} C^{j^z}_z\right)^{\frac{\eta^z-1}{\eta^z}} \\ +(1-n_{z1}-n_{z2})^{\frac{1}{\eta^z}}\left(\frac{1-n_{z1}-n_{z2}}{n_{z2}}\left(\frac{P_{z,z''}}{P_{z,z'}}\right)^{-\eta^z}\frac{n_{z2}}{n_{z1}}\left(\frac{P_{z,z'}}{P_{z,z}}\right)^{-\eta^z} C^{j^z}_z\right)^{\frac{\eta^z-1}{\eta^z}} \end{array}\right], \\
&= \left[\begin{array}{c} (n_{z1})^{\frac{1}{\eta^z}}\left(C^{j^z}_z\right)^{\frac{\eta^z-1}{\eta^z}} + (n_{z2})^{\frac{1}{\eta^z}}\left(\frac{n_{z2}}{n_{z1}}\right)^{\frac{\eta^z-1}{\eta^z}}\left(\frac{P_{z,z'}}{P_{z,z}}\right)^{-\eta^z\frac{\eta^z-1}{\eta^z}}\left(C^{j^z}_z\right)^{\frac{\eta^z-1}{\eta^z}} \\ +(1-n_{z1}-n_{z2})^{\frac{1}{\eta^z}}\left(\frac{1-n_{z1}-n_{z2}}{n_{z1}}\right)^{\frac{\eta^z-1}{\eta^z}}\left(\frac{P_{z,z''}}{P_{z,z}}\right)^{-\eta^z\frac{\eta^z-1}{\eta^z}}\left(C^{j^z}_z\right)^{\frac{\eta^z-1}{\eta^z}} \end{array}\right], \\
&= \left[\begin{array}{c} (n_{z1})^{\frac{1}{\eta^z}} + (n_{z2})^{\frac{\eta^z-1}{\eta^z}+\frac{1}{\eta^z}}\left(\frac{1}{n_{z1}}\right)^{\frac{\eta^z-1}{\eta^z}}\left(\frac{P_{z,z'}}{P_{z,z}}\right)^{1-\eta^z} \\ +(1-n_{z1}-n_{z2})^{\frac{\eta^z-1}{\eta^z}+\frac{1}{\eta^z}}\left(\frac{1}{n_{z1}}\right)^{\frac{\eta^z-1}{\eta^z}}\left(\frac{P_{z,z''}}{P_{z,z}}\right)^{1-\eta^z} \end{array}\right]\left(C^{j^z}_z\right)^{\frac{\eta^z-1}{\eta^z}}, \\
&= \left[\frac{(n_{z1})^{\frac{1}{\eta^z}}}{\left(\frac{1}{n_{z1}}\right)^{\frac{\eta^z-1}{\eta^z}}} + n_{z2}\left(\frac{P_{z,z'}}{P_{z,z}}\right)^{1-\eta^z} + (1-n_{z1}-n_{z2})\left(\frac{P_{z,z''}}{P_{z,z}}\right)^{1-\eta^z}\right]\left(\frac{C^{j^z}_z}{n_{z1}}\right)^{\frac{\eta^z-1}{\eta^z}}, \\
&= \left[(n_{z1})^{\frac{\eta^z-1}{\eta^z}+\frac{1}{\eta^z}} + n_{z2}\left(\frac{P_{z,z'}}{P_{z,z}}\right)^{1-\eta^z} + (1-n_{z1}-n_{z2})\left(\frac{P_{z,z''}}{P_{z,z}}\right)^{1-\eta^z}\right]\left(\frac{C^{j^z}_z}{n_{z1}}\right)^{\frac{\eta^z-1}{\eta^z}}, \\
&= \left[n_{z1} + n_{z2}\left(\frac{P_{z,z'}}{P_{z,z}}\right)^{1-\eta^z} + (1-n_{z1}-n_{z2})\left(\frac{P_{z,z''}}{P_{z,z}}\right)^{1-\eta^z}\right]\left(\frac{C^{j^z}_z}{n_{z1}}\right)^{\frac{\eta^z-1}{\eta^z}},
\end{aligned}
$$

$$
\begin{aligned}
P^{1-\eta^z}_{z,z}\left(C^{j^z}\right)^{\frac{\eta^z-1}{\eta^z}} &= \left[P^{1-\eta^z}_{z,z} n_{z1} + n_{z2}P^{1-\eta^z}_{z,z'} + (1-n_{z1}-n_{z2})\,P^{1-\eta^z}_{z,z''}\right]\left(\frac{1}{n_{z1}}\right)^{\frac{\eta^z-1}{\eta^z}}\left(C^{j^z}_z\right)^{\frac{\eta^z-1}{\eta^z}}, \\
C^{j^z} &= \frac{\left[P^{1-\eta^z}_{z,z} n_{z1} + n_{z2}P^{1-\eta^z}_{z,z'} + (1-n_{z1}-n_{z2})\,P^{1-\eta^z}_{z,z''}\right]^{\frac{\eta^z}{\eta^z-1}}}{P^{-\eta^z}_{z,z}}\left(\frac{1}{n_{z1}}\right)C^{j^z}_z,
\end{aligned}
$$

finally, by definition of price index, $P^{1-\eta^z}_z \equiv n_{z1}\left(P_{z,z}\right)^{1-\eta^z} n_{z2}\left(P_{z,z'}\right)^{1-\eta^z}(1-n_{z1}-n_{z2})\left(P_{z,z''}\right)^{1-\eta^z}$, it simplifies to $C^{j^z} = \frac{\left[P_z^{-(\eta^z-1)}\right]^{\frac{\eta^z}{\eta^z-1}}}{P^{-\eta^z}_{z,z}}\left(\frac{1}{n_{z1}}\right)C^{j^z}_z = \frac{P_z^{-\eta^z}}{P^{-\eta^z}_{z,z}}\left(\frac{1}{n_{z1}}\right)C^{j^z}_z$ and finally:

$$
C^{j^z}_z = n_{z1}\left(\frac{P_{z,z}}{P_z}\right)^{-\eta^z} C^{j^z}. \tag{83}
$$

where the denominator, is a trade-weighted domestic price index for z-country imports and home production. Replacing Equation (83) into $C^{j^z}_{z'} = \frac{n_{z2}}{n_{z1}}\left(\frac{P_{z,z'}}{P_{z,z}}\right)^{-\eta^z} C^{j^z}_z$, yields the imports of z from the z':

$$
C^{j^z}_{z'} = n_{z2}\left(\frac{P_{z,z'}}{P_z}\right)^{-\eta^z} C^{j^z},
$$

finally, the imports demand of z from z'' is given by:

$$C_{z''}^{j^z} = (1 - n_{z1} - n_{z2}) \left(\frac{P_{z,z''}}{P_z} \right)^{-\eta^z} C^{j^z}.$$

In a N-country model, there are N bilateral *net* trade flows; however, $N-1$ of them are independent because the world economy is "closed". Likewise, there are N bilateral terms of trade and N bilateral real exchange rates; however, only $N-1$ are independent. In our three country model set up, terms of trade are as follow:

$$T_{z,z'} \equiv \frac{P_{z,z'}}{P_{z,z}}, \qquad T_{z,z''} \equiv \frac{P_{z,z''}}{P_{z,z}}, \qquad T_{z',z''} \equiv \frac{P_{z,z''}}{P_{z,z'}} = \frac{T_{z,z''}}{T_{z,z'}}. \tag{84}$$

The same reasoning to get varieties' demands is applied (given the allocations $C_z^{j^z}$, $C_{z'}^{j^z}$ and $C_{z''}^{j^z}$). Let us define ζ, ζ', ζ'' between brackets denote a typical variety produced in countries z, z' and z'', respectively.

The optimization problem relevant for the domestic variety is:

$$\min_{C^{j^z}(\zeta)} \int_0^{n_z} p(\zeta) C^{j^z}(\zeta) d\zeta; \text{ st.: } C_z^{j^z} = \left[\left(\frac{1}{n_z} \right)^{\frac{1}{\varphi^z}} \int_0^{n_z} C^{j^z}(\zeta)^{\frac{\varphi^z - 1}{\varphi^z}} d\zeta \right]^{\frac{\varphi^z}{\varphi^z - 1}}.$$

Similarly, for imported varieties from country $z' \in Z^c$: $\min\limits_{C^{j^z}(\zeta')} \int_0^{n_{z'}} p_z(\zeta') C^{j^z}(\zeta') d\zeta'$; st.: $C_{z'}^{j^z} = \left[\left(\frac{1}{n_{z'}} \right)^{\frac{1}{\varphi^z}} \int_0^{n_{z'}} C^{j^z}(\zeta')^{\frac{\varphi^z - 1}{\varphi^z}} d\zeta' \right]^{\frac{\varphi^z}{\varphi^z - 1}}$, where $p_z(\zeta')$ is the price of the variety ζ' placed in country z (if the LOOP applies, then $p_z(\zeta') = S_{z,z'} p(\zeta')$ and $p_z(\zeta'') = S_{z,z''} p(z''))$. Let us solve these problems is turn. The first Lagrangian can be written as:

$$\mathcal{L} = \int_0^{n_z} p(\zeta) C^{j^z}(\zeta) d\zeta + \lambda_z \left\{ C_z^{j^z} - \left[\left(\frac{1}{n_z} \right)^{\frac{1}{\varphi^z}} \int_0^{n_z} C^{j^z}(\zeta)^{\frac{\varphi^z - 1}{\varphi^z}} d\zeta \right]^{\frac{\varphi^z}{\varphi^z - 1}} \right\},$$

which yield the following FOCs:

$$\mathcal{L}_{c(\zeta)} = 0 \Leftrightarrow p(\zeta) - \lambda_z \frac{\varphi^z}{\varphi^z - 1} \left(C_z^{j^z} \right)^{\frac{1}{\varphi^z - 1}} \left(\frac{1}{n_z} \right)^{\frac{1}{\varphi^z}} \frac{\varphi^z - 1}{\varphi^z} C^{j^z}(\zeta)^{\frac{-1}{\varphi^z}} = 0$$

$$\mathcal{L}_{c(\breve{\zeta})} = 0 \Leftrightarrow p(\breve{\zeta}) - \lambda_z \frac{\varphi^z}{\varphi^z - 1} \left(C_z^{j^z} \right)^{\frac{1}{\varphi^z - 1}} \left(\frac{1}{n_z} \right)^{\frac{1}{\varphi^z}} \frac{\varphi^z - 1}{\varphi^z} C^{j^z}(\breve{\zeta})^{\frac{-1}{\varphi^z}} = 0$$

$$\mathcal{L}_{\lambda_z} = 0 \Leftrightarrow C_z^{j^z} = \left[\left(\frac{1}{n_z} \right)^{\frac{1}{\varphi^z}} \int_0^{n_z} C^{j^z}(\zeta)^{\frac{\varphi^z - 1}{\varphi^z}} d\zeta \right]^{\frac{\varphi^z}{\varphi^z - 1}}$$

where $\breve{\zeta}$ is other variety that can be distinguished from ζ. Calculating the ratio $\mathcal{L}_{C(\zeta)} / \mathcal{L}_{C(\breve{\zeta})}$:

$$\frac{p(\zeta)}{p(\breve{\zeta})} = \frac{C^{j^z}(\zeta)^{\frac{-1}{\varphi^z}}}{C^{j^z}(\breve{\zeta})^{\frac{-1}{\varphi^z}}} \Leftrightarrow \frac{p(\zeta)}{p(\breve{\zeta})} = \frac{C^{j^z}(\zeta)^{\frac{-1}{\varphi^z}}}{C^{j^z}(\breve{\zeta})^{\frac{-1}{\varphi^z}}} \Leftrightarrow \frac{p(\zeta)}{p(\breve{\zeta})} C^{j^z}(\zeta)^{\frac{1}{\varphi^z}} = C^{j^z}(\breve{\zeta})^{\frac{1}{\varphi^z}} \Rightarrow C^{j^z}(\breve{\zeta}) = \left[\frac{p(\zeta)}{p(\breve{\zeta})} \right]^{\varphi^z} C^{j^z}(\zeta)$$

replacing $C^{j^z}(\breve{\zeta}) = \left[\frac{p(\zeta)}{p(\breve{\zeta})} \right]^{\varphi^z} C^{j^z}(\zeta)$ into the budget constraint $(\mathcal{L}_{\lambda_z} = 0)$, we get:

$$C_z^{j^z}=\left[\left(\tfrac{1}{n_z}\right)^{\frac{1}{\varphi^z}}\int_0^{n_z}\left(\left[\tfrac{p(\zeta)}{p(\breve{\zeta})}\right]^{\varphi^z}C^{j^z}(\zeta)\right)^{\frac{\varphi^z-1}{\varphi^z}}d\breve{\zeta}\right]^{\frac{\varphi^z}{\varphi^z-1}}=\left[\left(\tfrac{1}{n_z}\right)^{\frac{1}{\varphi^z}}\int_0^{n_z}\left(\tfrac{p(\zeta)^{\varphi^z}}{p(\breve{\zeta})^{\varphi^z}}\right)^{\frac{\varphi^z-1}{\varphi^z}}d\breve{\zeta}\right]^{\frac{\varphi^z}{\varphi^z-1}}C^{j^z}(\zeta)$$

$$=\left[\left(\tfrac{1}{n_z}\right)^{\frac{1}{\varphi^z}}p(\zeta)^{\varphi^z-1}\int_0^{n_z}\left(\tfrac{1}{p(\breve{\zeta})^{\varphi^z}}\right)^{\frac{\varphi^z-1}{\varphi^z}}d\breve{\zeta}\right]^{\frac{\varphi^z}{\varphi^z-1}}C^{j^z}(\zeta)=\left[\left(\tfrac{1}{n_z}\right)^{\frac{1}{\varphi^z}}\int_0^{n_z}p(\breve{\zeta})^{1-\varphi^z}d\breve{\zeta}\right]^{\frac{\varphi^z}{\varphi^z-1}}C^{j^z}(\zeta)p(\zeta)^{\varphi^z}$$

which, by aggregate price definition $(P_{z,z})^{1-\varphi^z}\equiv\left(\frac{1}{n_z}\right)^{\frac{1}{\varphi^z}}\int_0^{n_z}p(\breve{\zeta})^{1-\varphi^z}d\breve{\zeta}$, it follows that $C_z^{j^z}=(P_{z,z})^{-\varphi^z}C^{j^z}(\zeta)p(\zeta)^{\varphi^z}$; as a result, we obtain Equation (24) in the main text. Substituting it into the relationship $C^{j^z}(\breve{\zeta})=\left[\frac{p(\zeta)}{p(\breve{\zeta})}\right]^{\varphi^z}C^{j^z}(\zeta)$, yields $C^{j^z}(\breve{\zeta})$, but for symmetry and given the infinite varieties represented in the unit interval, its specification is similar to Equation (24) and redundant.

The second Lagrangian,

$$\mathcal{L}=\int_0^{n_{z'}}S_{z,z'}p(\zeta')C^{j^z}(\zeta')d\zeta'+\lambda_z\left\{C_{z'}^{j^z}-\left[\left(\frac{1}{n_{z'}}\right)^{\frac{1}{\varphi^z}}\int_0^{n_{z'}}C^{j^z}(\zeta')^{\frac{\varphi^z-1}{\varphi^z}}d\zeta'\right]^{\frac{\varphi^z}{\varphi^z-1}}\right\},$$

is solved in a similar way, and it yields the import's demand of the home consumer:

$$C^{j^z}(\zeta')=\frac{1}{n_{z'}}\left(\frac{p_z(\zeta')}{P_{z,z'}}\right)^{-\varphi^z}C_{z'}^{j^z},$$

where $(P_{z,z'})^{1-\varphi^z}\equiv\left(\frac{1}{n_{z'}}\right)^{\frac{1}{\varphi^z}}\int_{n_z}^{n_{z'}}p_z(\zeta')^{1-\varphi^z}d\zeta'$.

Proceeding in the same manner, the corresponding demand of varieties produced in country z'' is given by:

$$C^{j^z}(\zeta'')=\frac{1}{1-n_z-n_{z'}}\left(\frac{p_z(\zeta'')}{P_{z,z''}}\right)^{-\varphi^z}C_z^{j^z},$$

where $(P_{z,z''})^{1-\varphi^z}\equiv\left(\frac{1}{1-n_z-n_{z'}}\right)^{\frac{1}{\varphi^z}}\int_{n_z+n_{z'}}^{1}p_z(\zeta'')^{1-\varphi^z}d\zeta''$.

Likewise, from the point of view of country z' home and import demands are:

$$C^{j^{z'}}(\zeta')=\frac{1}{n_{z'}}\left(\frac{p(\zeta')}{P_{z',z'}}\right)^{-\varphi^{z'}}C_{z'}^{j^{z'}},\ C^{j^{z'}}(\zeta)=\frac{1}{n_z}\left(\frac{p_{z'}(\zeta)}{P_{z',z}}\right)^{-\varphi^{z'}}C_z^{j^{z'}},\ \text{and}$$
$$C^{j^{z'}}(\zeta'')=\frac{1}{1-n_z-n_{z'}}\left(\frac{p_{z'}(\zeta'')}{P_{z',z''}}\right)^{-\varphi^{z'}}C_{z''}^{j^{z'}}.$$

Finally from the point of view of country z'' home demand and imports are:

$$C^{j^{z''}}(\zeta'')=\frac{1}{1-n_z-n_{z'}}\left(\frac{p(\zeta'')}{P_{z'',z''}}\right)^{-\varphi^{z''}}C_{z''}^{j^{z''}},\ C^{j^{z''}}(\zeta)=\frac{1}{n_z}\left(\frac{p_{z''}(\zeta)}{P_{z'',z}}\right)^{-\varphi^{z''}}C_z^{j^{z''}},\ \text{and}$$
$$C^{j^{z''}}(\zeta')=\frac{1}{n_{z'}}\left(\frac{p_{z''}(\zeta')}{P_{z'',z'}}\right)^{-\varphi^{z''}}C_{z'}^{j^{z''}}.$$

A.2.2 Government consumption

The government consumers home goods solely. The Lagrangian is:

$$\mathcal{L}_z = \int_0^{n_z} p(\zeta)g(\zeta)d\zeta + \lambda_z \left\{ G_z - \left[\left(\frac{1}{n_z}\right)^{\frac{1}{\varphi^z}} \int_0^{n_z} g(\zeta)^{\frac{\varphi^z-1}{\varphi^z}} d\zeta \right]^{\frac{\varphi^z}{\varphi^z-1}} \right\},$$

which can be solved in a similar way as the intratemporal consumer problem, yielding the government's demand of variety ζ:

$$g(\zeta) = \frac{1}{n_z} \left(\frac{p(\zeta)}{P_{z,z}}\right)^{-\varphi^z} G_z.$$

A.2.3 Relative consumption price aggregates

In this section we briefly present relative consumption price aggregates of:

1. Country z:

$$\frac{P_z}{P_{z,z}} = T_{z,z'}^{n_{z2}} T_{z,z''}^{1-n_{z1}-n_{z2}}, \quad \frac{P_z}{P_{z,z'}} = T_{z,z'}^{n_{z2}-1} T_{z,z''}^{1-n_{z1}-n_{z2}}, \quad \frac{P_z}{P_{z,z''}} = T_{z,z''}^{n_{z2}-n_{z1}} T_{z,z'}^{-n_{z2}}.$$

2. Country z':

$$\frac{P_{z'}}{P_{z',z}} = T_{z',z''}^{1-n_{z'1}-n_{z'2}} T_{z',z}^{n_{z'2}-1}, \frac{P_{z'}}{P_{z',z'}} = T_{z',z}^{n_{z'2}} T_{z',z''}^{1-n_{z'1}-n_{z'2}}, \frac{P_{z'}}{P_{z',z''}} = T_{z',z}^{n_{z'2}}.$$

3. Country z'':

$$\frac{P_{z''}}{P_{z'',z'}} = T_{z'',z}^{n_{z''2}}, \frac{P_{z''}}{P_{z'',z}} = T_{z'',z'}^{1-n_{z''1}-n_{z''2}}, \frac{P_{z''}}{P_{z'',z''}} = T_{z'',z}^{n_{z''2}} T_{z'',z'}^{1-n_{z''1}-n_{z''2}}.$$

A.3 Firms' problem

A.3.1 Cost minimization (static)

The static cost minimization problem can be stated as the following Lagrangian (recall that the utilization rate is a choice variable for the consumer j^z, a upper index that we omit):

$$\mathcal{L}_{z,t} = R_{z,t}^k u_{z,t} K_{t-1}^{i^z} + W_{z,t} L_t^{i^z} + \lambda \left[Y_t^{i^z} - A_{z,t} \left(u_{z,t} K_{t-1}^{i^z}\right)^{\alpha^z} \left(L_t^{i^z}\right)^{1-\alpha^z} + FC_z \right]$$

$$\frac{\partial \mathcal{L}_{z,t}}{\partial K_{t-1}^{i^z}} = 0 \Leftrightarrow R_{z,t}^k u_{z,t} - \lambda \alpha^z A_{z,t} \left(u_{z,t} K_{t-1}^{i^z}\right)^{\alpha^z} \left(L_t^{i^z}\right)^{1-\alpha^z} \left(u_{z,t} K_{t-1}^{i^z}\right)^{-1} u_{z,t} = R_{z,t}^k u_{z,t} - \lambda \alpha^z \frac{Y_t^{i^z}}{K_{t-1}^{i^z}} = 0$$

$$\frac{\partial \mathcal{L}_{z,t}}{\partial L_t^{i^z}} = 0 \Leftrightarrow W_{z,t} - \lambda \left(1-\alpha^z\right) A_{z,t} \left(u_{z,t} K_{t-1}^{i^z}\right)^{\alpha^z} \left(L_t^{i^z}\right)^{1-\alpha^z} \left(L_t^{i^z}\right)^{-1} = W_{z,t} - \lambda \left(1-\alpha^z\right) \frac{Y_t^{i^z}}{L_t^{i^z}} = 0$$

solving the ratio $\left(\frac{\partial \mathcal{L}_{z,t}}{\partial K_{t-1}^{i^z}}\right) \left(\frac{\partial \mathcal{L}_{z,t}}{\partial L_t^{i^z}}\right)^{-1}$ yields:

$$\frac{R_{z,t}^k}{W_{z,t}} = \frac{\alpha^z}{\left(1-\alpha^z\right)} \frac{L_t^{i^z}}{u_{z,t} K_{t-1}^{i^z}}$$

that implies $K_{t-1}^{i^z} = \frac{W_{z,t}}{u_{z,t} R_{z,t}^k} \frac{\alpha^z}{(1-\alpha^z)} L_t^{i^z}$. The latter is substituted into Equation (40), that results from $\frac{d\mathcal{L}_{z,t}}{d\lambda} = 0$:

$$Y_t^{i^z} = A_{z,t} \left(\frac{W_{z,t}}{R_{z,t}^k} \frac{\alpha^z}{(1-\alpha^z)} L_t^{i^z} \right)^{\alpha^z} \left(L_t^{i^z} \right)^{1-\alpha^z} - FC_{z,t} = A_{z,t} \left(\frac{W_{z,t}}{R_{z,t}^k} \frac{\alpha^z}{(1-\alpha^z)} \right)^{\alpha^z} L_t^{i^z} - FC^{i^z},$$

$$L_t^{i^z} = \left(\frac{W_{z,t}}{R_{z,t}^k} \frac{\alpha^z}{(1-\alpha^z)} \right)^{-\alpha^z} \frac{\left(Y_t^{i^z} + FC^{i^z} \right)}{A_{z,t}}, \tag{85}$$

plugging the optimal demand for $L_t^{i^z}$ into $K_{t-1}^{i^z} = \frac{W_{z,t}}{u_{z,t} R_{z,t}^k} \frac{\alpha^z}{(1-\alpha^z)} L_t^{i^z}$, yields:

$$K_{t-1}^{i^z} = \frac{1}{u_{z,t}} \left(\frac{W_{z,t}}{R_{z,t}^k} \frac{\alpha^z}{(1-\alpha^z)} \right)^{1-\alpha^z} \frac{\left(Y_t^{i^z} + FC^{i^z} \right)}{A_{z,t}}. \tag{86}$$

These optimal input demands can be replaced back into the TC function:

$$TC_t^{i^z} = R_{z,t}^k \left(\frac{W_{z,t}}{R_{z,t}^k} \frac{\alpha^z}{(1-\alpha^z)} \right)^{1-\alpha^z} \frac{\left(Y_t^{i^z} + FC^{i^z} \right)}{A_{z,t}} + W_{z,t} \left(\frac{W_{z,t}}{R_{z,t}^k} \frac{\alpha^z}{(1-\alpha^z)} \right)^{-\alpha^z} \frac{\left(Y_t^{i^z} + FC^{i^z} \right)}{A_{z,t}},$$

operating:

$$TC_t^{i^z} = \left(R_{z,t}^k \right)^{\alpha^z} \left(W_{z,t} \right)^{1-\alpha^z} \left(\frac{\alpha^z}{(1-\alpha^z)} \right)^{1-\alpha^z} \frac{\left(Y_t^{i^z} + FC^{i^z} \right)}{A_{z,t}} + \left(R_{z,t}^k \right)^{\alpha^z} \left(W_{z,t} \right)^{1-\alpha^z} \left(\frac{\alpha^z}{(1-\alpha^z)} \right)^{-\alpha^z} \frac{\left(Y_t^{i^z} + FC^{i^z} \right)}{A_{z,t}},$$

factoring

$$TC_t^{i^z} = \left(R_{z,t}^k \right)^{\alpha^z} \left(W_{z,t} \right)^{1-\alpha^z} \frac{\left(Y_t^{i^z} + FC^{i^z} \right)}{A_{z,t}} \left[\left(\frac{\alpha^z}{(1-\alpha^z)} \right)^{1-\alpha^z} + \left(\frac{\alpha^z}{(1-\alpha^z)} \right)^{-\alpha^z} \right],$$

operating in the squared brackets:

$$\left(\frac{\alpha^z}{(1-\alpha^z)} \right)^{1-\alpha^z} + \left(\frac{\alpha^z}{(1-\alpha^z)} \right)^{-\alpha^z} = \left(\frac{\alpha^z}{(1-\alpha^z)} \right) + 1 = \frac{1}{(1-\alpha^z)}$$

replacing:

$$TC_t^{i^z} = \left(R_{z,t}^k \right)^{\alpha^z} \left(W_{z,t} \right)^{1-\alpha^z} \frac{\left(Y_t^{i^z} + FC^{i^z} \right)}{A_{z,t}} \left(\alpha^z \right)^{-\alpha^z} \left(1 - \alpha^z \right)^{\alpha^z - 1}$$

$$TC_t^{i^z} = \frac{\left(R_{z,t}^k \right)^{\alpha^z}}{\left(\alpha^z \right)^{\alpha^z}} \frac{\left(W_{z,t} \right)^{1-\alpha^z}}{\left(1 - \alpha^z \right)^{1-\alpha^z} A_{z,t}} \left(Y_t^{i^z} + FC^{i^z} \right). \tag{87}$$

The marginal cost can be derived from Equation (87) as $\partial TC_t^{i^z} / \partial Y_t^{i^z}$:

$$MC_t^{i^z} = \frac{\left(R_{z,t}^k \right)^{\alpha^z}}{\left(\alpha^z \right)^{\alpha^z}} \frac{\left(W_{z,t} \right)^{1-\alpha^z}}{\left(1 - \alpha^z \right)^{1-\alpha^z} A_{z,t}}, \tag{88}$$

where MC_t does not depend on firm i^z's output; therefore, we drop the upper index in the main text.

A.3.2 Intertemporal problem: Calvo pricing

As stated in the main text, the profit of the home firm that serves the domestic market is given by Equation (46), subject an aggregated version of the domestic demand Equation (45), that firm i^z must serve. Moreover, the discount factor that should be taken into account is derived form the FOC w.r.t. to home bonds as follows: $E_t\left[R^{-1}_{z,t+h}\right] = \beta^h E_t\left[\Lambda^1_{z,t+h}\right]/\Lambda^1_{z,t}$; (the envelope theorem assures that all returns at the optimum are the same, so we disregards foreign bonds because they convey the same information). Replacing we can rewrite profits to be maximized as:

$$\mathsf{Pr}^{i^z}_t = E_t\left[\sum_{h=0}^{\infty}\left(\alpha^z_P\beta\right)^h \frac{\Lambda^1_{z,t+h}}{\Lambda^1_{z,t}}\left[\left(1-\tau^z_I\right)\mathcal{S}^z_P\tilde{p}^{i^z}_t(\zeta)Y^{i^z}_{z,t+h}(\zeta) - TC^{i^z}_{z,t+h}\left(Y^{i^z}_{z,t+h}(\zeta)+FC_z\right)\right]\right] \tag{89}$$

from where we derive the following FOC:[56]

$$E_t\left[\begin{array}{c}\sum\limits_{h=0}^{\infty}\left(\alpha^z_P\beta\right)^h \frac{\Lambda^1_{z,t+h}}{\Lambda^1_{z,t}} \\ \times \\ \left[\begin{array}{c}(1\text{-}\varphi^z)\,(1\text{-}\tau^z_I)\,\mathcal{S}^z_P\left(\left(\frac{\tilde{p}^{i^z}_t(\zeta)}{P_{z,z,t+h}}\right)^{-\varphi^z}\left(\frac{n_{z1}}{n_z}\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{-1}(C_{z,t+h}+I_{z,t+h})+\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{-\varphi^z}G_{z,t+h}\right)\right) \\ +\varphi^z MC_{z,t+h}\left(\frac{\tilde{p}^{i^z}_t(\zeta)}{P_{z,z,t+h}}\right)^{-\varphi^z-1}\frac{1}{P_{z,z,t+h}}\left(\frac{n_{z1}}{n_z}\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{-1}(C_{z,t+h}+I_{z,t+h})+\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{-\varphi^z}G_{z,t+h}\right)\end{array}\right]\end{array}\right]$$

equalizing it to zero, introducing the summation and the expectations operator, and then sending the first term to the RHS given the fact that $1-\varphi^z = -\left(\varphi^z-1\right)$, yields:

$$\frac{(\varphi^z-1)\left(1-\tau^z_I\right)\mathcal{S}^z_P}{\Lambda^1_{z,t}}\tilde{p}^{i^z}_t(\zeta)^{-\varphi^z}E_t\left[\sum_{h=0}^{\infty}\left(\alpha^z\beta\right)^h\Lambda^1_{z,t+h}\left(P^{\varphi^z}_{z,z,t+h}\left(\frac{n_{z1}}{n_z}\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{-1}(C_{z,t+h}+I_{z,t+h})+\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{-\varphi^z}G_{z,t+h}\right)\right)\right.$$
$$=\varphi^z\frac{\left(\tilde{p}^{i^z}_t(\zeta)\right)^{-\varphi^z}}{\tilde{p}^{i^z}_t(\zeta)\Lambda^1_{z,t}}E_t\left[\sum_{h=0}^{\infty}\left(\alpha^z_P\beta\right)^h\Lambda^1_{z,t+h}MC_{t+h}P^{\varphi^z}_{z,z,t+h}\left(\frac{n_{z1}}{n_z}\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{-1}(C_{z,t+h}+I_{z,t+h})+\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{-\varphi^z}G_{z,t+h}\right)\right].$$

Finally the optimal price quoted domestically to buy the home variety ζ, $\tilde{p}^{i^z}_t(\zeta)$, is equal to:

$$\frac{\mu^z}{\left(1-\tau^z_I\right)\mathcal{S}^z_P}\frac{E_t\left[\sum\limits_{h=0}^{\infty}\left(\alpha^z_P\beta\right)^h\Lambda^1_{z,t+h}MC_{t+h}P^{\varphi^z}_{z,z,t+h}\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{-1}\left(\frac{n_{z1}}{n_z}(C_{z,t+h}+I_{z,t+h})+\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{1-\varphi^z}G_{z,t+h}\right)\right]}{E_t\left[\sum\limits_{h=0}^{\infty}\left(\alpha^z_P\beta\right)^h\Lambda^1_{z,t+h}P^{\varphi^z}_{z,z,t+h}\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{-1}\left(\frac{n_{z1}}{n_z}(C_{z,t+h}+I_{z,t+h})+\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{1-\varphi^z}G_{z,t+h}\right)\right]}, \tag{90}$$

where we define the relevant markup as $\mu^z \equiv \frac{\varphi^z}{(\varphi^z-1)}$ and $\mathcal{S}^z_P$ stands for a lump-sum subsidy, when greater than one, that exactly neutralizes the markup. This is an artifact to replicate an scenario where the monopolistic distortion is corrected by the government employing fiscal policy.

To derive the (relative) **real optimal price**, consider $\tilde{p}^{i^z}_t(\zeta)$ given by Equation (90) and define its real version as $\widetilde{rp}^{i^z}_t(\zeta) \equiv \frac{\tilde{p}^{i^z}_t(\zeta)}{P_{z,t}}$. Given the fact that $\Lambda^1_{z,t+h} = \tilde{\Lambda}^1_{z,t+h}/P_{z,t+h}$ (this allows us to pin down real marginal costs in the numerator and gross inflation in the denominator, since it is multiplied by $P_{z,t}$), yields:

$$\frac{\mu^z}{\left(1-\tau^z_I\right)\mathcal{S}^z_P}\frac{E_t\left[\sum\limits_{h=0}^{\infty}\left(\alpha^z_P\beta\right)^h\tilde{\Lambda}^1_{z,t+h}mc_{t+h}P^{\varphi^z-1}_{z,z,t+h}P_{z,t+h}\left(\frac{n_{z1}}{n_z}(C_{z,t+h}+I_{z,t+h})+\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{1-\varphi^z}G_{z,t+h}\right)\right]}{P_{z,t}E_t\left[\sum\limits_{h=0}^{\infty}\left(\alpha^z_P\beta\right)^h\frac{\tilde{\Lambda}^1_{z,t+h}}{P_{z,t+h}}P^{\varphi^z-1}_{z,z,t+h}P_{z,t+h}\left(\frac{n_{z1}}{n_z}(C_{z,t+h}+I_{z,t+h})+\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{1-\varphi^z}G_{z,t+h}\right)\right]},$$

[56] Note that our Cobb-Douglas production function leads to a marginal cost function Equation (88) that does not depend on output, so that MC_t has no arguments.

where inside the numerator and the denominators' summations, multiplying and dividing by $(P_{z,t+h})^{\varphi^z-1}$

$$\frac{\mu^z}{\left(1-\tau_I^z\right)\mathcal{S}_P^z}\frac{E_t\left[\sum_{h=0}^{\infty}\left(\alpha_P^z\beta\right)^h\tilde{\Lambda}_{z,t+h}^1 mc_{t+h}P_{z,z,t+h}^{\varphi^z-1}\frac{\left(P_{z,t+h}\right)^{\varphi^z-1}}{\left(P_{z,t+h}\right)^{\varphi^z-1}}\Pi_{z,t+h}\left(\frac{n_{z1}}{n_z}\left(C_{z,t+h}+I_{z,t+h}\right)+\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{1-\varphi^z}G_{z,t+h}\right)\right]}{E_t\left[\sum_{h=0}^{\infty}\left(\alpha_P^z\beta\right)^h\tilde{\Lambda}_{z,t+h}^1 P_{z,z,t+h}^{\varphi^z-1}\frac{\left(P_{z,t+h}\right)^{\varphi^z-1}}{\left(P_{z,t+h}\right)^{\varphi^z-1}}\left(\frac{n_{z1}}{n_z}\left(C_{z,t+h}+I_{z,t+h}\right)+\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{1-\varphi^z}G_{z,t+h}\right)\right]}$$

$$\frac{\mu^z}{\left(1-\tau_I^z\right)\mathcal{S}_P^z}\frac{P_{z,t}^{1-\varphi^z}E_t\left[\sum_{h=0}^{\infty}\left(\alpha_P^z\beta\right)^h\tilde{\Lambda}_{z,t+h}^1 mc_{t+h}\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{\varphi^z-1}P_{z,t+h}^{\varphi^z-1}\Pi_{z,t+h}\left(\frac{n_{z1}}{n_z}\left(C_{z,t+h}+I_{z,t+h}\right)+\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{1-\varphi^z}G_{z,t+h}\right)\right]}{P_{z,t}^{1-\varphi^z}E_t\left[\sum_{h=0}^{\infty}\left(\alpha_P^z\beta\right)^h\tilde{\Lambda}_{z,t+h}^1\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{\varphi^z-1}P_{z,t+h}^{\varphi^z-1}\left(\frac{n_{z1}}{n_z}\left(C_{z,t+h}+I_{z,t+h}\right)+\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{1-\varphi^z}G_{z,t+h}\right)\right]}$$

$$\widetilde{rp}_t^{i^z}(\zeta)=\frac{\mu^z}{\left(1-\tau_I^z\right)\mathcal{S}_P^z}\frac{E_t\left[\sum_{h=0}^{\infty}\left(\alpha_P^z\beta\right)^h\tilde{\Lambda}_{z,t+h}^1 mc_{t+h}\Pi_{z,t+h}^{\varphi^z}\left(\frac{n_{z1}}{n_z}\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{\varphi^z-1}\left(C_{z,t+h}+I_{z,t+h}\right)+G_{z,t+h}\right)\right]}{E_t\left[\sum_{h=0}^{\infty}\left(\alpha_P^z\beta\right)^h\tilde{\Lambda}_{z,t+h}^1\Pi_{z,t+h}^{\varphi^z-1}\left(\frac{n_{z1}}{n_z}\left(\frac{P_{z,z,t+h}}{P_{z,t+h}}\right)^{\varphi^z-1}\left(C_{z,t+h}+I_{z,t+h}\right)+G_{z,t+h}\right)\right]} \tag{91}$$

Before advancing in the exporters sectors' pricing, notice that country z' demands for imports from countries z and z'' are $C^{j^{z'}}(\zeta)=\frac{1}{n_z}\left(\frac{p_{z'}(\zeta)}{P_{z',z}}\right)^{-\varphi^{z'}}C_z^{j^{z'}}$ and $C^{j^{z'}}(\zeta'')=\frac{1}{1-n_z-n_{z'}}\left(\frac{p_{z'}(\zeta'')}{P_{z',z''}}\right)^{-\varphi^{z'}}C_{z''}^{j^{z'}}$, respectively. Likewise, for country z'', imports from country z are $C^{j^{z''}}(\zeta)=\frac{1}{n_z}\left(\frac{p_{z''}(\zeta)}{P_{z'',z}}\right)^{-\varphi^{z''}}C_z^{j^{z''}}$ and from country z': $C^{j^{z''}}(\zeta')=\frac{1}{n_{z'}}\left(\frac{p_{z''}(\zeta')}{P_{z'',z'}}\right)^{-\varphi^{z''}}C_{z'}^{j^{z''}}$. Moreover, prices for imported goods in country z are:

$$\left(P_{z,z'}\right)^{1-\varphi^z}\equiv\left(\frac{1}{n_{z'}}\right)^{\frac{1}{\varphi^z}}\int_0^{n_{z'}}p_z^{1-\varphi^z}(\zeta')d\zeta',$$

$$\left(P_{z,z''}\right)^{1-\varphi^z}\equiv\left(\frac{1}{1-n_z-n_{z'}}\right)^{\frac{1}{\varphi^z}}\int_{n_z+n_{z'}}^{1}p_z^{1-\varphi^z}(\zeta'')d\zeta''.$$

For simplicity in the main text we made the assumption that the LOOP applies so that the price paid in the foreign country of a home good is just the domestic price multiplied by the relevant nominal exchange rate. This implies that what applied between braces both in the numerator and denominator of (91) can be replaced by $Y_{z,t+h}^{i^z}(\zeta)+FC_z$ which comes from the equilibrium condition (supply equals demands, domestic and foreign). Of course, $Y_{z,t+h}^{i^z}(\zeta)$ is taken from the production function (40).

A.4 Wages dispersion

To derive the nominal average wage dispersion, consider Equation (62) in the main text. It follows that:

$$\begin{aligned}
\tilde{\Delta}_{W,z,t} &= L_{z,t}^{1+\sigma_L^z}\left(\frac{1}{n_z}\right)\int_0^{n_z}\left(\frac{W_t^{j^z}}{W_{z,t}}\right)^{-\sigma_W^z\left(1+\sigma_L^z\right)} dj^z \\
&= L_{z,t}^{1+\sigma_L^z}\left(\frac{1}{n_z}\right)\int_0^{n_z}\left[\sum_{h=0}^{\infty}\left(1-\alpha_W^z\right)\left(\alpha_W^z\right)^h\left(\frac{\tilde{W}_{t-h}^{j^z}}{W_{z,t}}\right)^{-\sigma_W^z\left(1+\sigma_L^z\right)}\right] dj^z, \\
&= L_{z,t}^{1+\sigma_L^z}\left(1-\alpha_W^z\right)\left(\frac{1}{n_z}\right)\int_0^{n_z}\left[\sum_{h=0}^{\infty}\left(\alpha_W^z\right)^h\left(\frac{\tilde{W}_{t-h}^{j^z}}{W_{z,t}}\right)^{-\sigma_W^z\left(1+\sigma_L^z\right)}\right] dj^z, \\
&= L_{z,t}^{1+\sigma_L^z}\left(1-\alpha_W^z\right)\sum_{h=0}^{\infty}\left(\alpha_W^z\right)^h\left[\left(\frac{1}{n_z}\right)\int_0^{n_z}\left(\frac{\tilde{W}_{t-h}^{j^z}}{W_{z,t}}\right)^{-\sigma_W^z\left(1+\sigma_L^z\right)} dj^z\right],
\end{aligned}$$

integrating over j^z yields:

$$\tilde{\Delta}_{W,z,t} = L_{z,t}^{1+\sigma_L^z}\left(1-\alpha_W^z\right)\underbrace{\sum_{h=0}^{\infty}\left(\alpha_W^z\right)^h\left(\frac{\tilde{W}_{z,t-h}}{W_{z,t}}\right)^{-\sigma_W^z\left(1+\sigma_L^z\right)}}_{\equiv\Delta_{W,z,t}},$$

expanding the summation, $\Delta_{W,z,t}$:

$$\Delta_{W,z,t} = \left(\frac{\tilde{W}_{z,t}}{W_{z,t}}\right)^{-\sigma_W^z\left(1+\sigma_L^z\right)} + \sum_{h=1}^{\infty}\left(\alpha_W^z\right)^h\left(\frac{\tilde{W}_{z,t-h}}{W_{z,t}}\right)^{-\sigma_W^z\left(1+\sigma_L^z\right)},$$

the lower index in the summation is lagged one period and we multiply the second term by $\left(\frac{W_{z,t-1}}{W_{z,t-1}}\right)^{-\sigma_W^z\left(1+\sigma_L^z\right)}$, so that it yields:

$$\begin{aligned}
\Delta_{W,z,t} &= \left(\frac{\tilde{W}_{z,t}}{W_{z,t}}\right)^{-\sigma_W^z\left(1+\sigma_L^z\right)} + \left(\frac{W_{z,t-1}}{W_{z,t}}\right)^{-\sigma_W^z\left(1+\sigma_L^z\right)}\sum_{h=0}^{\infty}\left(\alpha_W^z\right)^{h+1}\left(\frac{\tilde{W}_{z,t-h-1}}{W_{z,t-1}}\right)^{-\sigma_W^z\left(1+\sigma_L^z\right)}, \\
&= \left(\frac{\tilde{W}_{z,t}}{W_{z,t}}\right)^{-\sigma_W^z\left(1+\sigma_L^z\right)} + \alpha_W^z\left(\frac{W_{z,t-1}}{W_{z,t}}\right)^{-\sigma_W^z\left(1+\sigma_L^z\right)}\sum_{h=0}^{\infty}\left(\alpha_W^z\right)^{h}\left(\frac{\tilde{W}_{z,t-h-1}}{W_{z,t-1}}\right)^{-\sigma_W^z\left(1+\sigma_L^z\right)}, \\
&= \left(\breve{w}_{z,t}\right)^{-\sigma_W^z\left(1+\sigma_L^z\right)} + \alpha_W^z\Pi_W^{\sigma_W^z\left(1+\sigma_L^z\right)}\Delta_{W,z,t-1}
\end{aligned}$$

since we employ definitions of $\breve{w}_{z,t}\equiv\frac{\tilde{W}_{z,t}}{W_{z,t}}$ and Π_W. Replacing $\Delta_{W,z,t}$ into $\tilde{\Delta}_{W,z,t}$ above yields:

$$\tilde{\Delta}_{W,z,t} = L_{z,t}^{1+\sigma_L^z}\left(1-\alpha_W^z\right)\Delta_{W,z,t}.$$

A.5 Prices dispersion

Likewise, we can derive prices dispersion and finally obtain Equation (56). We omit this derivation as it is parallel as for wage dispersion. Here we aim at obtaining the real version of (56). To do so, we multiply and divide it by $P_{z,t}$ (when necessary) to get:

$$\Delta_{z,t} = \left(\frac{P_{z,t}}{P_{z,t}}\right)^{-\varphi^z} \left(P_{z,z,t}\right)^{\varphi^z} \left(1-\alpha_P^z\right) \left(\widetilde{p}_t^{i^z}(\zeta)\right)^{-\varphi^z} + \alpha_P^z \left(\frac{P_{z,z,t}}{P_{z,z,t-1}} \frac{P_{z,t}}{P_{z,t}} \frac{P_{z,t-1}}{P_{z,t-1}}\right)^{\varphi^z} \Delta_{z,t-1}, \qquad (92)$$

where by employing relative prices definitions yields:

$$\Delta_{z,t} = \left(\frac{P_{z,z,t}}{P_{z,t}}\right)^{\varphi^z} \left(1-\alpha_P^z\right) \left(\widetilde{rp}_t^{i^z}(\zeta)\right)^{-\varphi^z} + \alpha_P^z \left(\frac{p_{z,z,t}}{p_{z,z,t-1}} \Pi_{z,t}\right)^{\varphi^z} \Delta_{z,t-1},$$

gives the real version of Equation (56) in the main text, where we assumed that there are no exporters.